

On the Integration of Computer Algebra Systems (CAS) by Canadian Mathematicians: Results of a National Survey

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Abstract: In this article, we outline the findings of a Canadian survey study (N = 302) that focused on the extent of computer algebra systems (CAS)-based technology use in postsecondary mathematics instruction. Results suggest that a considerable number of Canadian mathematicians use CAS in research and teaching. CAS use in research was found to be the strongest factor affecting CAS integration in teaching. Mathematicians believe that CAS is becoming an integral part of contemporary mathematics knowledge. Two main factors impeding CAS integration are the departmental culture and the time required for designing CAS-based resources. Mathematicians mostly incorporate CAS use into assignments and much less for in-class tests and final examinations. CAS integration in teaching appears to remain a predominantly individual initiative.

Résumé: Dans cet article, nous traçons les grandes lignes des résultats d'une étude canadienne (N = 302) ayant pour objet l'utilisation des technologies fondées sur les systèmes de calcul formel en enseignement des mathématiques au niveau post-secondaire. Les résultats indiquent qu'un nombre considérable de mathématiciens canadiens utilisent les systèmes de calcul formel dans leur recherche et leur enseignement. Il ressort également que leur utilisation en recherche est le facteur le plus important pour ce qui est de l'intégration des systèmes de calcul formel en enseignement. Les mathématiciens estiment que les systèmes de calcul formel constituent désormais une partie intégrante des connaissances mathématiques contemporaines. Deux facteurs principaux en gênent cependant l'intégration : la culture générale des départements et le temps nécessaire pour créer des ressources fondées sur ces systèmes. En général, les mathématiciens se servent des systèmes de calcul formel surtout dans les tâches, et beaucoup moins dans les évaluations en classe et dans les examens de fin de session. L'intégration de ces systèmes dans l'enseignement demeure largement le fait d'initiatives personnelles.

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INTRODUCTION

In its most recent *Curriculum Guide* (Mathematics Association of America [MAA], 2004), *Part I: Recommendations for Departments, Programs, and All Courses*, the MAA's Committee on the Undergraduate Program in Mathematics made the following recommendation, as part of the introduction to "Section 5: Use of Computer Technology to Support Problem Solving and to Promote Understanding": "At every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology: appropriately and effectively as a tool for solving problems; and, as an aid to understanding mathematical ideas" (p. 22).

The average North American mathematician might wonder how much of this recommendation is actually being implemented by departments of mathematics, beyond her or his own institutional context. Furthermore, she or he might be interested in knowing whether there has been research conducted to measure success in the area of technology implementation and, if so, upon which criteria it was based. Are younger, or early-career, mathematicians more inclined to integrate digital technology in their teaching than their older or more established faculty counterparts?

Some 25 years prior to the Curriculum Guide, the International Commission on Mathematics Instruction (ICMI) in their Study 1 (Churchhouse et al., 1986), The Influence of Computers and Informatics on Mathematics and Its Teaching, projected that in the years to come a wide integration of digital technologies in mathematics education would take place. But in 2006, the ICMI Study 17 (Hoyles & Lagrange, 2010), in revisiting this topic two decades later, stressed that the promising integration of technology in mathematics education had not yet been fully realized. Whereas the first ICMI Study had focused mainly on university-level instruction, the ICMI Study 17 was almost exclusively concentrated on school-level education. This current trend is also reflected in the literature regarding technology use in mathematics education, which overwhelmingly focuses on school-level education (Lavicza, 2008b). What, then, is there to say about the current technology integration in postsecondary mathematics instruction? The ICMI Study 11 (Holton, 2007), The Teaching and Learning of Mathematics at University Level, dedicated a section to this topic (Section 5: Technology). Somewhat surprisingly, in contrast, the ICMI Study 14 (Galbraith, Henn, & Niss, 2007), Modeling and Applications in Mathematics Education, did not address the rich context of technology use in university education. In his international comparative research, Lavicza (2008b) found that university mathematicians use digital technology at least as much as school teachers and that age and years of service were not significantly correlated to computer algebra system (CAS)-based technology use in university teaching. Based on his study, Lavicza (2008b) recommended that the innovative teaching practices involving technology that are already implemented by mathematicians should be more fully researched and documented.

In this article, we outline the findings of a Canadian survey study, which was designed as an extension of Lavicza's (2008a) study, regarding the extent of CAS-based technology use in postsecondary mathematics instruction. CAS-based technology is a digital technology that combines capabilities for symbolic, numeric, and graphic computation (e.g., Maple, Mathematica, Derive). Like most technology, CAS technology is constantly evolving, and most CAS-based software programs now have user-friendly graphical user interfaces. Lavicza (2008a) noted the following statement regarding CAS: [CAS] have a distinguished position among [Information and Communication Technology] applications employed in mathematics education (Allen et al., 1999; Kendal & Stacey, 2001). CAS can be generally utilized in the teaching and learning of a variety of mathematics topics. Furthermore, CAS has the potential to become a powerful instrument in a student's mathematical toolkit. Noss and Hoyles's (2003) characterization of mathematical software packages also illustrates the prominent role of CAS among other educational technology applications. (pp. 48–49)

Lavicza further explained how CAS was developed initially as a powerful computational and visualization tool for mathematicians but was soon after adopted for instructional purposes. Furthermore, "It is argued that achieving proficiency in the use of such complex mathematical packages may influence students' future studies and professional work as CAS remains a powerful computational instrument for solving emerging mathematical problem" (Lavicza, 2008a, p. 49). Whereas the applicability of software such as dynamic geometry systems (e.g., The Geometer's Sketchpad, Cabri), which was initially developed for teaching purposes (Noss & Hoyles, 2003), is restricted to a narrower range of topics, CAS packages are more extensive and lend themselves to a wider range of uses in various mathematical fields (Lavicza, 2008b). In summary, CAS can be considered the most widely used and prominent software packages used at the university level (Allen et al., 1999; Lavicza, 2008a). For this reason, and because the aim of the Canadian research was to extend Lavicza's existing study, our study primarily focused on this particular technology.

In the next section of this article, we describe the research methodology. Then the findings are presented and discussed followed by concluding comments. This study was part of a broader research program that aimed at both documenting postsecondary teaching practices involving digital technology and formulating recommendations for individual and departmental change. The research program involved three main research components: (a) an extensive literature review focusing on CAS use in postsecondary mathematics instruction, which was based on a pilot literature review (Buteau, Marshall, Jarvis, & Lavicza, 2010); (b) a case study of two departments that have succeeded in implementing a systemic integration of digital technologies in their core undergraduate mathematics programs (Jarvis, Lavicza, & Buteau, forthcoming); and (d) the Canadian survey study of CAS integration in postsecondary mathematics teaching that is being reported herein. Complementing these three components of our research program, a workshop aiming at fostering discussion among Canadian mathematicians about the integration of digital technologies in postsecondary mathematics instruction was organized and hosted at two national research institutes in Fall 2010, namely, at the Centre de recherche mathématique in Montreal and at the Fields Institute for Research in Mathematical Sciences in Toronto (Buteau, Jarvis, & Lavicza, 2011). At these venues, the preliminary results of the survey study were presented and, in addition to a keynote, three panel discussions addressed key issues of technology use in postsecondary instruction. The later three are brought up and tied to the survey results in the Results and Discussion.

METHODOLOGY

This survey study was designed to obtain a preliminary picture of CAS integration in Canadian postsecondary mathematics instruction. Overall the study addressed the following three issues: (1) the extent of CAS use in postsecondary institutions; (b) views of mathematicians on the role of

CAS in mathematics literacy and curriculum; and (c) factors influencing the integration of CAS into mathematics teaching and learning at the postsecondary level. The study was also designed to compare the results with those of a similar international survey study conducted in the United States, United Kingdom, and Hungary (Lavicza, 2008a). In fact, Lavicza became part of the research team for the Canadian study and, as such, this research can be viewed as a form of direct extension of his international survey study, albeit with data collection taking place 2 years later. We therefore used a similar methodology (participant recruitment, questionnaire, and analytical tools) as in Lavicza's (2008a) research but with slight modifications to the survey (e.g., Canadian context question modifications, French translation for Francophone participants) where deemed appropriate.

Lavicza's (2008a) comprehensive study followed a two-phase, mixed-methods approach, for which the first qualitative phase involved interviews, class observations, and the review of curriculum materials of 22 mathematicians from the three participating countries. The second phase, mostly quantitative, was dedicated to the survey, which was sent to 4,500 mathematicians in the three countries. Based on the results of the first phase and supported by the literature (e.g., social-cultural theories as used by Ruthven and Hennessy [2002] and Becker [2001]), Lavicza developed an online questionnaire over a period of 7 months (many pilot versions preceded the final version). The final questionnaire included 35 main questions (a 5-point Likert scale with an N/A option, where applicable, and a few optional open-ended questions) among which many involved subquestions. Survey questions inquired about mathematician participants' background, views on CAS in mathematics teaching and learning, access to technology and training, and their own use of CAS in teaching, as well as soliciting open responses for several questions (Lavicza, 2010). The robustness of the questionnaire was confirmed by the statistical analysis. The survey data were imported into SPSS software and descriptive and inferential statistical techniques were performed, as well as structural equation modeling using AMOS software. Written responses (open-ended questions) were analyzed using a grounded theory approach, constant comparative analysis, and repeated coding (Strauss & Corbin, 1998).

For the Canadian survey study, we slightly adapted the methodology to take into account the Canadian reality. This meant that the survey questionnaire¹ first needed to be prepared in both official languages, English and French.² In addition, because the educational system is structured differently in the province of Quebec, where a 2-year college instruction period (College of General and Vocational Education, or CÉGEP³) precedes a normally 3-year university degree (compared to normally 4 years in Anglophone universities), we decided to include the CÉGEPs in our survey. Although there were several issues that warranted further investigation and/or changes in wording to the survey, we decided to align the Canadian survey items very closely with Lavicza's (2008a) survey for the sake of direct comparisons. Furthermore, to increase participation we did not want to extend the length of the time (approximately 20 minutes) needed to complete the survey (Gunn, 2002). A decision was therefore made to insert only two additional questions relating to the use of broader technologies in research and teaching.

The analysis of the data was guided by Lavicza's similar analysis of his international survey data. Descriptive analysis was performed on all of the Canadian data, as well as inferential analysis for comparing results from CAS users and non-CAS users. In addition, an exploratory factor analysis was conducted to identify characteristics of mathematicians who indicated that they do integrate CAS in their teaching. The latter method

is a statistical technique that can be used for exploring latent variable structures in datasets (Field, 2005). [Exploratory Factor Analysis] techniques are based on the identification of clusters of highly correlated variables. . . . [It] enables researchers to reduce the data to a small number of explainable concepts while accounting for the maximum amount of common variance. (Lavicza, 2008a, p. 202)

For the comparison with international trends, this was done in two ways: (a) treating Canada as an additional (i.e., fourth) participating country and (b) contrasting Canadian results directly with the combined international results. In this article, we focus the discussion only on Canadian trends, with the intent to communicate the international survey comparison results in a subsequent publication. The written responses were analyzed with the same method as for the international study.

Nearly 2000 personalized e-mails were sent to all mathematicians listed in official websites of 60 departments of mathematics in Canadian universities, plus one set of subsequent personalized e-mail reminders. For joint departments (e.g., departments of Mathematics and Computer Science), we decided to send e-mails to all instructors because it was not always possible to easily distinguish the main discipline of listed instructors. It also meant that mathematics didacticians working in departments of mathematics received the invitation e-mail, whereas those working in faculties of education did not. For CÉGEP mathematics instructors, the same personalized e-mail method could not be applied because most CÉGEP websites do not explicitly list all of their instructors by discipline. We therefore used a mailing list (54 of 56 public and private CÉGEPS, which included 170 instructors) through their main professional association, Association de mathématique du Québec. Personalized e-mails were all bilingual, with the French translation appearing at the top part of the e-mail for mathematicians working in the province of Quebec (including the generic e-mail to CÉGEP instructors) and the English version (i.e., English at the top part) used for all other mathematicians. Both versions (French and English) of the online survey were located on Free Online Survey system.⁴ Eventually, 302 mathematicians responded to the online survey, 223 to the English version and 79 to the French version, with a total response rate of about⁵ 14.5%.

The Survey Participants

The demography of the survey respondents was as follows: the majority were male (80%) and full-time employees (90%). Their age was almost uniformly distributed (age <35, 36–45, 46–55, 56–65), although there was a minority (6%) of senior instructors over the age of 65. The largest group regarding years of teaching experience was that of professors who had taught at least 20 years (40%), whereas the smallest group was that of new professors with only 1–3 years of experience (6%), with the remaining categories (experience 4–7, 8–12, 13–20 years) being equally distributed. Most mathematician participants in our survey declared their primary research field as "pure mathematics," closely followed by the category of "applied mathematics." Indeed, 46% of Anglophones⁶ identified themselves as pure mathematicians, and 12% as statisticians, whereas 25% of Francophones⁷ identified themselves as pure mathematics educators. Twenty-seven percent of Francophone participants indicated that they did not conduct research as part of their regular work, a result that can be explained by the inclusion of CÉGEP instructors who are not usually expected to conduct research as part of their contract.

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Challenges With the Sampling and Questionnaire

Some issues regarding the sampling procedures need to be mentioned. The decision to include both the CÉGEP mathematics instructors and the university mathematics educators from mathematics departments might seem questionable. Because mathematics educators from mathematics departments were included in the other countries surveyed by Lavicza (e.g., in Hungary where mathematics didacticians are usually working in departments of mathematics), we decided not to exclude them in our study for the sake of coherence with the international survey (Lavicza, 2008a). As for mathematics instructors strictly teaching at CÉGEPs (14.6% of participants), we decided to include them to better cohere with the 4-year undergraduate programs in Anglophone Canada. It is clear that their main responsibility is teaching, as it is for lecturers working in university departments also included in our survey and, as such, influences the reported percentages but not the overall trends.

An unfortunate issue with the URL in the personalized invitation e-mails sent to all mathematicians working in a Canadian department of mathematics may have somewhat negatively impacted the participation rate in the survey study. A shortened URL link⁸ had been used for concision but the alternative URL used a free Hungarian link that was in some cases detected as spam by university computer servers, or was perceived as spam by mathematicians themselves. In addition, the provincial origin of participants was not tracked, participants volunteered for the study in a self-selected manner, and the sample's representativeness of all Canadian mathematicians remains unknown. The conclusions of the study are thus somewhat limited by these various sampling issues.

In terms of the questionnaires, some questions asked about teaching practices without separating courses in the wording of the statements, making it difficult for participants to accurately answer some of those questions. In addition, some questions were missing the "N/A" (not applicable) option, as per the international survey wording, which we were replicating, and thus we were required to analyze the data for these particular questions with extra caution.⁹.

RESULTS AND DISCUSSION

Findings of the survey study are presented according to seven topics:

- 1. The extent of CAS use by Canadian mathematicians in teaching and research
- 2. Mathematicians' views on the role of CAS in mathematical literacy and teaching
- 3. Factors potentially hindering the integration of CAS in teaching
- 4. Diverse uses of CAS in teaching
- 5. CAS use in assessment
- 6. CAS integration and departmental culture
- 7. Integration of other digital technologies in teaching

Extensive Use of CAS in Research and Teaching

A noteworthy majority of respondents (69%) reported using CAS in their teaching (see Figure 1, left graph). An even larger number of participants (81%) reported using CAS in their research (see



FIGURE 1 To the left, Use of CAS in Teaching: "In a typical academic term, in approximately what percentage of your lessons do you use CAS?"; to the right, Use of CAS in Research "In an average working month, how frequently do you use CAS in your research?"

Figure 1, right graph). Even when taking into account certain biases¹⁰ inherent to the study, these figures are remarkably high and at least comparable with the technology use in school mathematics education. For example, in a recent study (Mullis, Martin, Gonzalez, & Chrostowski, 2004), the level of considerable computer use in Grade 8 classes from participating countries, including Canada, was reported to be occurring in only 3–5% of the classes.



FIGURE 2 Diagram Summarizing the Exploratory Factor Analysis (EFA) Regarding the Use of CAS in Teaching by Use of a Structural Equation Modeling (SEM) Regression Model, With the Observed Variables: Language (French, English), Age, Gender, Research Area, Integration of CAS in Research, and, Employment Status (chi-square = 0.000; df = 0; p = 0; comparative fit index = 1.000; root mean square error of approximation = 0.127). *Note*. The higher the value assigned to the observed variable (e.g., 0.45 for CAS use in research), the more causal effect it has on the CAS use in teaching variable. Values assigned to the round arrows on the left of the diagram represent covariance coefficients. (color figure available online)

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Interestingly, when we consider the reported use of CAS in research and teaching practices, 31% of all respondents indicated that they do not use CAS in their teaching, whereas only 19% do not use it in their research. According to our factor analysis (see Figure 2), the use of CAS in research is the strongest factor (r = 0.45; $r^2 = 0.2025$) affecting CAS integration in teaching, a result that corroborated the similar finding (r = 0.3; $r^2 = 0.09$) of Lavicza (2008a) in his survey study involving mathematics instructors in Hungary, the United Kingdom, and the United States. One survey respondent discussing the reasons to have started using CAS in his/her teaching, noted, "Because it was part of how I did certain parts of my research," and another commented on CAS as "one of my most important tools of research. I find it important to share this." Indeed, this result may in fact contribute to our understanding of why technology seems to be more integrated in postsecondary mathematics instruction than in schools (Lavicza, 2010). For many mathematicians, the use of certain digital technologies does not need to be learned to be used separately because the technologies are already a regular part of their research agenda and practice. However, this situation is often different for schoolteachers, in that they often require initial training with the software prior to using it in their teaching. By way of corollary, the mathematician participants who reported never using CAS in their research all mentioned either never (67%) or rarely¹¹ (33%) using this technology in their teaching.

In terms of graduate-level mathematics instruction, results similar to that of overall university instruction were observed: a majority of master's and PhD instructors (72 and 68%, respectively; see Figure 3) reported using CAS. But a closer look at the extent of use in each mathematics instruction level, and at mathematical use of CAS in research, leads to the conclusion that the use of CAS in graduate instruction is even more closely tied to the use of CAS in research than the results found at the undergraduate teaching level (see Figure 3). One survey participant commented, "I find CAS valuable for investigating open questions (research, projects) but less useful for clarifying basic mathematical concepts and acquiring basic skills. ... I have used



CAS use in different teaching levels and

FIGURE 3 Comparing CAS Use Extent in Undergraduate Mathematics University Teaching, Master's Teaching, PhD Teaching, and in Mathematical Research. In these bar graphs: in research, we assigned *frequently* to be least once a week; in teaching, we assigned it to be frequently or always.



FIGURE 4 To the left, the Evolution of CAS Use in Teaching Over the Years: "Has your use of CAS in your teaching increased over the years?"; to the right, the Year Experience of CAS Use in Teaching by CAS User Respondents "How many years have you been using CAS in your teaching?"

CAS successfully in undergraduate research projects, but rarely in a teaching environment." This supports the idea that CAS can be used by instructors in providing mathematical research-related experiences to undergraduate students (Lavicza, 2010).

Mathematicians' reported use of CAS in their teaching, over time, is also of interest. According to our survey, CAS use seems sustainable, because only 13% of CAS users indicated a less frequent use of CAS in their teaching as time passed in the role (see Figure 4, left graph). When asked how long they had been using CAS in teaching the results were well spread out, with a slim majority (56%) having used CAS in teaching between 4 and 12 years (see Figure 4, right graph).

Views on the Role of CAS in Mathematical Literacy and Curriculum

When asked about their views on the role of technology in the mathematics curriculum, and in light of mathematical literacy, respondents overall reported a positive stance. This aligns well with the findings from the international survey study in which Lavicza (2010) noted, "[T]he analysis of the data revealed that the majority of mathematicians believed that CAS is becoming an important element of mathematical literacy and will become an integral part of mathematics teaching and learning in the future" (p. 111).

Survey participants were asked to respond to eight statements related to mathematical literacy, which included, for example, "CAS is changing the way in which mathematics research is done"; see Table 1 for a complete list of statements including a summary of descriptive statistics. Among these statements, one in particular received an unusually high standard deviation ($\bar{x} = 2.88$; SD = 1.10),¹² suggesting a possible controversial statement: "CAS use does not affect the mathematics that has to be learned by students in post-secondary institutions." It is also true that for this unique statement, the position of CAS user and non-CAS user respondents did not differ (see Table 1). Mathematicians do not seem to agree as to the impact of CAS on mathematical knowledge. In addition, 37% of mathematician respondents disagreed with the statement that "knowing how to use CAS is an essential skill for mathematics graduates," yet only 13% disagreed with the statement, "CAS enables mathematicians to work on problems more efficiently." Above all, the

TABLE 1

Literacy Variable Comparison by Users and Nonusers of CAS in Teaching. The Mann-Whitney Tests Indicate Significant Differences (p < .5) for All Statements Between Users and Nonusers of CAS, Except for "CAS Use Does Not Affect the Mathematics That Has to be Learned by Students in Universities"

Mathematicians' views on the role of CAS in mathematics literacy										
Statement	All			Nonusers			Users			
	Ν	Mean	SD	Ν	Mean	SD	Ν	Mean	SD	Mann-Whitney U
CAS part of the curriculum Science and engineering graduates should have a working knowledge of CAS (+)	286	4.02	0.856	83	3.48	0.929	187	4.26	0.734	4,123.5 (<i>p</i> = .000)
Knowing how to use CAS is an essential skill for mathematics graduates (+)	286	3.85	1.115	83	3.05	1.219	187	4.22	0.851	3,452.0 (<i>p</i> = .000)
Knowing how to use CAS is beneficial for students in science and engineering courses (+)	285	4.27	0.716	83	3.80	0.777	187	4.46	0.589	4,137.5 (<i>p</i> = .000)
Knowing how to use CAS enhances students' future employment prospects (+)	285	3.71	0.887	82	3.35	0.961	187	3.85	0.816	5,466.5 (<i>p</i> = .000)
CAS changes research CAS is changing the way in which mathematics research is done (+)	284	3.68	0.940	82	3.30	1.015	186	3.84	0.878	5,311.0 (<i>p</i> = .000)
CAS enables mathematicians to work on problems more efficiently (+) CAS changes the curriculum	285	4.00	0.845	83	3.47	0.846	186	4.24	0.736	3,972 (<i>p</i> = .000)
CAS use does not affect the mathematics that has to be learned by students in postsecondary institutions (-)	282	2.88	1.104	80	2.85 (3.15)	0.982	186	2.89 (3.11)	1.169	7,357.5 (<i>p</i> = .882)
CAS offers the possibility of introducing new topics into undergraduate mathematics (+)	286	3.65	0.893	83	3.17	0.935	187	3.89	0.782	4,487.5 (<i>p</i> = .000)

results regarding these three statements point to the need of further epistemological discussions among mathematicians, as was stressed in the international survey study (Lavicza, 2010):

[M]athematicians accept that CAS is part of the literacy, but at the same time they are reluctant to accept that CAS shapes mathematical knowledge. This disparity is possibly derived from the mismatch between mathematicians' CAS-related and mathematical beliefs ... a closer examination of th[e] relationship between these conceptions would be beneficial. (p. 111)

The analysis of the data revealed that, overall, respondents viewed CAS-assisted teaching and learning in a positive light (see Table 2). Furthermore, among the 10 statements relating to potentially higher student motivation, aid to students' understanding, and engaging, interesting, interactive lessons, only two statements turned out to be possibly controversial, again based on standard deviation spread: "CAS use encourages students to examine carefully the meaning of their solutions" ($\bar{x} = 3.04$; SD = 1.08) and "CAS use does not help students to understand mathematical concepts" $\bar{x} = 2.58$; SD = 1.05). Not surprisingly, respondents who used CAS in their teaching place a considerably higher value on the role of CAS in teaching than those who do not use CAS in teaching (see Table 2). When CAS-user mathematicians were asked to elaborate on the principles that guide their CAS-related teaching, two ideas dominated: (1) CAS as a tool; for example, "CAS is a tool, not a purpose in itself. It has about the same ranking as a slide-rule would have had"; and (b) CAS use to support student understanding; for example, "I use CAS only when I believe it will help the students understand better."

Factors That May Hinder the Integration of CAS Into Teaching

Several potential factors that may hinder the integration of CAS into teaching were proposed in the survey. Mathematician respondents mostly agreed (54%) in identifying as a factor the lack of enthusiasm shown by colleagues toward using CAS in mathematics classes, as well as the time required for developing CAS-related teaching material (43%). A slightly less convincing result (37%) was the mention of limited class time making it difficult to add CAS-related activities. Syntax, large entry-level classes, cost of the technology, use of CAS in assessment, and poor user-friendliness of the software were not flagged as substantial factors. However, issues of accessibility were often pointed out: whereas the technology was reported as being available for use in most lecture rooms, and it was accessible for everyday use, many mathematicians (30%) thought that the number of computer labs in their department was not sufficient for CAS-assisted teaching. Interestingly, a larger proportion (54%) of Francophone mathematicians agreed that it is not difficult to schedule a mathematics class in a computer lab than did their Anglophone counterparts (27%). Somewhat ironically, this proportion is reversed when questioned about the availability of technical support to those lecturers who need it.

Mathematicians were invited to further elaborate on their views of factors hindering CAS integration in teaching and, in particular, non-CAS users were also given the opportunity to briefly explain what their reasons were for not using CAS technology in teaching. Two ideas dominated: time and relevance. Mathematicians reported the following time-related concerns, not only toward themselves but also in relation to students and to teaching: time to develop CAS-related teaching material; for example, "The primary hindrance is the amount of time it takes to develop really good CAS materials ... you have to do it altruistically in most cases"; limited time in the classroom; for example, "Major factor: if I was to use CAS, I must have more

TABLE 2

CAS Teaching and Learning Variable Comparison by Users and Nonusers of CAS in Teaching. The Mann-Whitney Tests Indicate Significant Differences (p < .5) for All Statements Between Users and Nonusers of CAS

	Mathematicians' views on CAS-assisted teaching and learning									
Statement	All			Nonusers			Users			
	Ν	Mean	SD	Ν	Mean	SD	Ν	Mean	SD	Nonusers/users
Student learning CAS use helps students to develop better understanding of mathematical concepts (+)	284	3.39	0.909	81	2.93	0.891	188	3.63	0.814	4,382.0 (<i>p</i> = .000)
CAS use does not help students to understand mathematical concepts (-)	286	2.58	1.052	83	3.07 (2.93)	1.010	188	2.35 (3.65)	0.978	4,807.5 (<i>p</i> = .000)
CAS use encourages students to examine carefully the meaning of their solutions (+)	286	3.04	1.075	83	2.61	0.961	188	3.23	1.059	$5,324.0 \ (p = .000)$
CAS use distracts students from understanding mathematical concepts (-)	285	2.59	0.988	83	3.01 (2.99)	0.956	187	2.39 (3.61)	0.935	5,128.5 (<i>p</i> = .000)
Teaching—Motivation Images generated by CAS improve students' attention in class (+)	285	3.49	0.794	82	3.17	0.663	188	3.71	0.718	4,488.5 (<i>p</i> = .000)
CAS enables teachers to deliver more engaging lessons (+)	284	3.47	0.919	83	2.90	0.806	187	3.72	0.854	3894.0 (<i>p</i> = .000)
CAS use does not make classes more interesting for students (-)	285	2.73	0.864	83	3.14 (2.86)	0.798	187	2.52 (3.48)	0.844	4,676.0 (<i>p</i> = .000)
CAS use has positive effects on students' enthusiasm for mathematics (+)	286	3.34	0.820	83	3.07	0.777	188	3.46	0.823	5,817.0 (<i>p</i> = .000)
Teaching—Communication CAS-generated images spark valuable discussions in class (+)	285	3.49	0.794	83	3.08	0.702	187	3.68	0.763	4,634.5 (<i>p</i> = .000)
CAS use can initiate in-class communication between students (+)	284	3.31	0.781	83	3.12	0.705	186	3.42	0.789	6,225.5 (<i>p</i> = .003)

time scheduled—cannot do it within constraints of time and syllabus as is"; time for students to learn to use a software; for example, "It takes time for the students to learn how to use the CAS, time which they need to learn the basic mathematical concepts"; and time for a mathematician to learn the software if not already proficient; for example, "Not interested and I would have to spend time learning it." Assude, Buteau, and Forgasz (2010) pointed to the additional pressure (e.g., with time, expertise, bureaucracy) on mathematicians as a factor impeding the integration of digital technologies in university teaching.

Many mathematicians also indicated that CAS technology is not the most relevant technology with regard to the courses that they teach; for example, some mention other technologies that are more specifically designed for statistics courses. Another reason was pointing at the nonrelevance of CAS-based technology to help conceptual understanding and the need to first teach basic skills. Furthermore, some indicated the lack of evidence showing the relevance and benefits of its use in teaching; for example, "I haven't seen a convincing reason for its use" and "There is nothing stopping me from using it. I just don't see how it helps students learn mathematics". In their literature review, Buteau et al. (2010) found that issues regarding pedagogy (e.g., assessment, possible failure of students to achieve learning objectives, and students' overreliance on CAS which might degrade students' abilities in absence of the tool) were by far the most discussed concerns (by CAS-user respondents) as barriers to integration of CAS technology. Financial and technical barriers were also identified as barriers but to a lesser degree.

Mathematicians discussed several other factors, such as computer and software accessibility issues (for students and instructors). Departmental support and working conditions have influenced some mathematicians in their decision to integrate, or not, CAS in teaching; for example, "For the large intro classes, the decision is not mine to make" or "No one is forcing me." And for some, the more traditional chalk and blackboard approach to instruction simply remains unquestionable: "Blackboard explanation in class is the best way to teach."

Diverse Uses of CAS in Instruction

CAS-based technologies can be used in mathematics teaching in many different ways. Three different uses were reported as most common practice, in each case by about 90% of CAS-user respondents: (a) to assign projects and homework to students to work at home; (b) for student experimentation with CAS; and (c) for visualization and illustration purposes (see Figure 5). But in each of these three cases, approximately half of the CAS-user respondents indicated an occasional use only. A slightly less common use (73% of CAS users) is the development of course materials that encourage students to work with CAS-based technologies. One of the potential benefits of integrating digital technologies into a course is to enrich the traditional lecture format, with more interactive, student-centered activities (Wolfram, 2010). For example, approximately half of the CAS-user respondents to work in teams during lectures. The development of CAS-based worksheets by mathematicians for their students to use in class was used by a similar percentage of respondents. The development of online tutorials was shown to be less common (37%).

In addition, computer-based CAS technology seems more common (80%) than handheld CAS devices (20%). CAS-user mathematicians integrated the CAS technology most often in homework and projects (84%) but also in lecture rooms (71%) and computer labs (67%).



Different ways of using CAS

FIGURE 5 The Different Uses of CAS in Instruction (reordered according to the participants' extent of use): "In what way do you use CAS in your teaching? I use CAS to (a) project images to illustrate concepts; (b) encourage students to experiment with CAS; (c) encourage students to work in teams/groups in lectures; (d) assign project/homework for students to work at home; (e) develop worksheets for students to work with; (f) develop online tutorials for students; (g) develop course materials that encourage students to work with CAS."

A mathematician participant commented, "It is not whether I use CAS, but which kind, when, where, and for who!" Indeed, our analysis suggests that the extent of CAS use differs for different student groups (see Figure 6). Mathematician CAS-user respondents overwhelming (92%) indicated using CAS for mathematics and computer science majors, with a remarkable



CAS use by major

FIGURE 6 CAS Use With Different Majors: "With what kinds of student groups do you use CAS in your teaching? Courses designed for:"

proportion (47%) of frequent (or always) use. This large proportion could be partially explained by the considerable use of CAS in graduate mathematics teaching. A relatively smaller proportion of CAS users (87–89%) indicated using CAS when teaching science or engineering majors. Furthermore, a remarkably smaller proportion (70%) of respondents used CAS for educational majors, with half of this group using it only occasionally. We contend that this finding could potentially contribute to our understanding of why there appears to be a low integration of CAS-based technology (and other digital technologies) in school education.

Use of CAS in Assessment

Related to the diverse uses of CAS-based technologies in instruction is its use in assessment. A large majority (94%) of CAS-user respondents indicated the use of CAS in homework and assignment course work (see Figure 7, top). However, only 22% of all CAS-user respondents integrated CAS, at least occasionally, in final exams, and only 27% in classroom tests. Interestingly, whereas only 37% of all respondents do not believe that "it is difficult to assess what students know if they use CAS in tests" (see Figure 7, bottom), only 19%¹³ actually do use CAS in their in-class tests.

At first sight, one might argue that instructor beliefs do not align very closely with instructor practice. But there are distinctions in terms of use that need to be taken into consideration when commenting on the extent of CAS use in summative assessment within a CAS-integrated course. For our discussion, we distinguish three different types of instructional purposes, or goals, of CAS-based technology use. CAS technology can be used as a learning tool for students; for example, for visualizing concepts, exploring cases, etc., possibly with the intent of an instructor to increase student motivation or support students' deeper conceptual understandings. In this case, the question whether to use, or not use, CAS in summative assessment is irrelevant. A similar argument holds when the CAS-based technology is used as a teaching tool by the instructor; for example, for preparing homework material, illustrating concepts in the classroom, assessing students online, etc.

However, when the instructor's intention is to have students use CAS as a tool to do mathematics—for example, to solve realistic, complex problems; to analyze multiple representations of concepts; etc.—then the question of whether or not to integrate CAS technology in summative assessment is crucial (Caron & Ben-El-Mechaiekh, 2010). There are a number of challenges when adopting technology for summative assessment activities. For example, communication among students during in-class assessment is found to be problematic (e.g., one participant commented, "Unfortunately, we have found that increasing technology use (particularly during exams) seems to encourage students to cheat. Why this is, I don't know."), as is the time required for instructors to modify traditional exam questions. In fact, in their literature review, Buteau et al. (2010) observed that assessment was the integration issue the most discussed by CAS-user instructors.

CAS Integration and Departmental Culture

Though departments do not appear to discourage CAS use, and though CAS seems readily available for use in many departments, the choice to use CAS appears to remain at the discretion of individual instructors; that is, not compulsory (see Figure 8). As was similarly highlighted

15% 10% 5%



Use of CAS in Assessment

 0%
 strongly
 disagree
 neutral
 agree
 strongly agree
 N/A

 FIGURE 7
 On top, Use of CAS in Assessment: "Do you permit CAS to be used during assessments?" On bottom,

Mathematicians' Views on Difficulty in Integrating CAS in Assessment: "It is difficult to assess what students know if they can use CAS in tests."

(Marshall, Buteau, Jarvis, & Lavicza, 2012) in the international survey (Lavicza, 2008a), this finding corresponds well to the results of the literature review by Buteau et al. (2010), which noted that only 6% of the reported CAS-based technology integration by practitioners indicated a program-wide or systemic implementation. In contrast, 67% of the reported use was linked to use in a single course or, in other words, by a single practitioner.

Part of the discussion here hinges on academic freedom, which allows an instructor to decide to use or not use CAS within a given course. On the one hand, this can be viewed as beneficial and positive, because the individual instructor is at liberty to innovate within a course or decide



FIGURE 8 Mathematicians' Perceptions of Departmental Approval to Use CAS in Teaching. To the left, "I can freely choose whether or not I use CAS in my teaching"; to the right, "My department does not encourage the use of CAS in math classes."

to integrate digital technologies in certain self-selected ways (Lavicza, 2006). But on the other hand, what could be perceived as one's freedom to integrate CAS or not may in fact limit the freedom of other colleagues to integrate the technology. Indeed, how much freedom is left when an instructor, teaching an upper-year course, cannot assume a basic level of computer literacy (e.g., CAS use knowledge or programming skills) from students that may be required in order to integrate meaningful CAS activities related to the topics covered in his/her course? Though the learning curve required for the competent use of a CAS can be perceived as costly in terms of any individual course, if this investment of time and energy is viewed as being spread over multiple courses and years, it can be perceived as much more worthwhile. A survey participant commenting on factors impeding CAS integration in teaching mentioned, "The learning curve on a CAS is high. I cannot lose all of my class time on its use if I plan to cover something else especially if I won't see the student again and they may never use it again." Another mathematician noted, "Too many choices … would be nice to have one standard [software]."

In fact, the MAA Curriculum Guide (MAA, 2004), in Part I: Recommendations for Departments, Programs, and All Courses, recommends that

In order for technology to be useful in mathematics instruction, students should be able to focus on the mathematics rather than on how to use the technology. ... Using the same software in several different courses also can shorten the total technology learning curve. (p. 23)

In addition to this issue of limited available time during courses, there is also the issue of timeconsuming preparation of material when integrating CAS in a course. This was indeed identified as a factor that may impede integration as mentioned previously. One respondent commented, "Too much time is required to develop useful and interesting examples." This could possibly be addressed with better collaboration between colleagues within a department, as highlighted by two different respondents: "[A factor hindering CAS integration is the] lack of collegial support among colleagues" and "Lack of initiative at the departmental level. The lack of willingness to standardize the use of such technology ... but it will change—the calculators are becoming too



Interest to collaborate in course development involving CAS

FIGURE 9 Mathematicians' Interest in Collaborating With Their Colleagues in the Development of Courses That Involve CAS Use: "I would be happy to collaborate with my colleagues to develop courses that involve CAS use."

powerful to ignore."¹⁴ In fact, many CAS-user respondents indicated an interest in collaborating with their colleagues in the development of courses that involve CAS use (see Figure 9).

Integration of Other Technologies in Teaching

Two questions relating to the use of broader technology in teaching and research (see Figure 10) were added to the questionnaire for the Canadian survey. CAS is, in our study, without a doubt the



FIGURE 10 Use of Diverse Technologies for Doing/Learning/Assessing Mathematics: "What kinds of technology have you used in your own research/teaching? Check all that apply: CAS (Maple, Mathematica, ...); programming (Java, C++, Fortran, ...); simulation software (Swarm, MCell, Simulink, ...); statistical analysis software (SAS, Minitab, Microsoft Excel, ...); geometry software (Cabri, Geometer's SketchPad, GeoGebra, ...); discrete mathematics systems (MATLAB, Microsoft Excel, ...); online assessment tools with personalized questions and automatic correction (Maple TA, WebCT, self-designed tool, ...); other (please specify)."

most prominent mathematics technology used in research and in teaching at the postsecondary level. This fact supported the choice of restricting the survey study to mostly CAS use. An interesting observation from our data is that a large majority (93%) of mathematician participants who are active in research use at least some technologies (among those listed in Figure 10; i.e., excluding communication technologies, such as e-mails, text editors, LaTeX, online forums, etc.) in their research work. In addition, 85% of mathematician participants reported using at least one technology (among those listed in Figure 10) in their teaching.

Except for computer programming, all different forms of technology used in mathematics teaching and research work are integrated approximately to the same extent (see Figure 10).¹⁵ This naturally raises the question as to why programming is relatively absent in mathematics teaching (note that it is used over twice as often in research, 43%, as it is in teaching, 18%). One could argue that the learning curve for students' use of programming is much steeper than for many other technologies that feature more user-friendly graphical interfaces, including most CAS (which can be used in many cases without involving any programming). Delisle, van Rensburg, and St-Aubin (2010) highlighted difficulties of a logistical nature when integrating a mathematics programming course in an undergraduate mathematics degree program. For example, who should teach the programming course or unit, who would want to teach such a course, and when should it be taught within the program? Further, there is also the challenge of its systemic integration *in practice*; that is, the use of programming in other courses.

According to our study (Figure 10), computer programming is the second most used mathematics technology in research, and this interesting fact points to the potential of such a tool for actually doing mathematics at an exploratory, computational, or application level. Whiteley, Heffernan, and Saliola (2010) discussed the issue of revising Ontario undergraduate mathematics curricula in light of university undergraduate degree-level expectations (Ontario Council of Universities, 2008) and the need of a deeper understanding of the integration of technology in mathematics program objectives and mathematics curriculum. In this context, reflection by mathematicians on the potential benefits of incorporating computer programming into mathematics research activities could, we feel, lead to an increased integration of computer programming in undergraduate mathematics instruction.

CONCLUDING COMMENTS

In this article, we have presented the findings of a Canadian-wide, online survey study regarding the use of CAS-based technologies in postsecondary mathematics education. The results suggest that an impressive number of Canadian mathematicians appear to use CAS and other digital technologies in their research and undergraduate and graduate teaching. Except for computer programming, all different forms of technology (i.e., used specifically for doing mathematics, and not including communication technology) used by Canadian mathematicians in mathematics teaching and research work seem to be integrated relatively to the same extent. CAS use in research emerges as the strongest factor affecting CAS integration in teaching, a result that corroborates a similar finding from Lavicza's (2008a) international study (the United States, the United Kingdom, and Hungary). In addition, our study suggests that "[mathematicians] believe that CAS is becoming an integral part of contemporary mathematics knowledge" (Lavicza, 2010, p. 112).

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The two main factors impeding CAS integration in teaching identified by the Canadian mathematicians appear to be the departmental culture (lack of enthusiasm by colleagues, lack of departmental support, etc.) and the time required for designing meaningful CAS-based activities and resources. Mathematicians seem to use CAS technology in their teaching mostly for assigning projects and homework (i.e., behind the scenes), for visualization purposes, and for having students experiment. In terms of assessment, a majority of Canadian CAS-user mathematicians appear to incorporate CAS use into homework assignments and projects, in contrast to a much smaller proportion of instructors who use CAS for in-class tests and final examinations. CAS integration seems to remain, by and large, a predominantly individual initiative. However, many mathematicians also indicate their interest in collaborating with colleagues to develop courses and resources that would involve CAS use.

Our Canadian findings do serve to complement and reinforce those of the international survey study (Lavicza, 2008a). Indeed, our overall results regarding CAS use by mathematicians from the Canadian study are very similar to those from the international survey study (Lavicza, 2008a) and, as such, suggest similar conclusions. In addition, the Canadian study also asked participants about the use of different digital technologies in teaching and research. The results point to two particularly interesting observations: (a) CAS seems to be the most used mathematics technology by Canadian mathematicians in teaching and in research and (b) the second most used technology by Canadian mathematicians in their research seems to be programming technology, whereas this technology seems to be integrated in teaching to a relatively much smaller extent as do other mathematics technologies.

The notable extent of CAS integration in teaching by Canadian mathematicians observed in our study is actually a phenomenon that is well supported by diverse organizations within their respective communities. For example, the Canadian Mathematics Education Study Group (CMESG) has devoted many working groups to the topic of technology, including some with a particular focus on postsecondary education. Among the most recent examples would be the CMESG 2012 working group entitled Technology and Mathematics Teachers (K-16) (Buteau & Sinclair, 2013); see also at CMESG 2011,¹⁶ 2008,¹⁷ and many others. In addition, the biannual conference of the Canadian Mathematical Society (CMS) normally devotes one session to a mathematics education theme, and some have focused on technology; for example, the 2008 CMS Winter conference session, entitled Technology Use in Post-Secondary Mathematics Instruction, which was documented in a report published in the official CMS newsletter education column (Jarvis, Buteau, & Lavicza, 2009). The Fields Institute for Mathematical Research in Toronto hosts a monthly mathematics Education Forum that regularly involves technology in its program; for example, the recent February 2012 forum entitled The Role of Technology in Assessment and Evaluation of Mathematics Learning. Finally, the Waterloo, Ontario-based company, MapleSoft, is among the world's leaders in CAS-based technology.

Future research that would complement this study might involve the surveying of mathematicians regarding their integration in teaching of (broader) technologies as a tool for students to do mathematics. This could, for example, include an examination of mathematicians' views on the potential benefits of students learning through mathematics computer programming.

The MAA Curriculum Guide (MAA, 2004) mentions the following:

[W]hen misused, technology can become a crutch, used for tasks that students should be able to do by hand and providing only an illusion of accomplishment. The reality of such problems with technology

should not cause mathematics departments to avoid its use. Rather, they indicate the care and effort needed for effective implementation. The potential benefits of technology for student learning are worth this care and effort. (pp. 23–24)

Our study suggests that a considerable number of Canadian mathematicians have integrated digital technologies in their teaching, while being aware of, and concerned about, their inherent challenges. As one of the experienced survey participants light-heartedly stressed: "I worked as a mathematician for [many] years in research and industry. I KNOW what a mathematician needs to know to be useful to society (i.e., have a job). Either you know how to use a computer, or you flip burgers." We believe that the potential benefits of technology use in mathematics teaching and learning—apart from employment, engagement, or even mere enjoyment factors—are indeed worth the effort of mathematicians as they further integrate technology in postsecondary mathematics instruction.

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NOTES

- 1. See Jarvis, n.d., http://casresearch.nipissingu.ca/canadiansurvey.htm to view the questionnaire.
- 2. The original questionnaire from the international study was in English for the United States and the United Kingdom and in Hungarian for Hungary.
- 3. From the French version, Collèges d'enseignement général et professionnel (CÉGEPs).
- 4. http://freeonlinesurveys.com/ (Free Online Surveys, n.d.).
- 5. Due to the URL that was used, some e-mails were wrongly identified by institutional servers as spam, and therefore some mathematicians did not receive the invitation.
- 6. In this survey, we use *Anglophone* for a survey participant who completed the English version of the survey and likewise use *Francophone* to denote those completing the survey in French. However, we are aware that this is not necessarily a faithful representation of the mother tongue or working tongue of the respondents. See the Discussion section for further details.
- 7. See note 6.

- 8. The survey read, "Take the CAS Survey (in English): http://href.hu/x/8dsv"; and, "Compléter le sondage (en français): http://href.hu/x/8dsw."
- 9. In some cases we could still use the data by restricting the participant groups identified through other questions; for example, the use of CAS in MSc instruction by mathematicians teaching MSc courses.
- 10. For example, the sampling was not random, because the whole target population was invited to participate, with approximately 15% volunteering to participate in the study.
- 11. Twenty-five percent or less of their lessons.
- 12. Recall that a Likert scale was used: 1 = strongly disagree to 5 = strongly agree.
- 13. When considering all survey participants, 27% of CAS-user participants yielded 19% of the sample.
- 14. Translated statement: "Le manque d'initiative au niveau départemental. Le manque de volonté de normaliser l'utilisation de tels technologie . . . mais ça va changer—les calculatrices deviennent trop puissantes pour ignorer."
- 15. Because online assessment tools are solely used in teaching, this statement comparing the use in teaching and in research does not pertain to this particular technology.
- 16. Etchecopar, Muller, and Villeneuve's working group: Using Simulation to Develop Students' Mathematical Competencies: Post-secondary and Teacher Education.
- 17. Buteau, Etchecopar, and Gadanidis's working group: Communication and Mathematical Technology Use throughout the Post-secondary Curriculum.

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