Computer Algebra System (CAS) Usage and Sustainability in University Mathematics Instruction: Findings from an International Study

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Abstract

This paper reports on research findings of Jarvis, Buteau, and Lavicza (2012) regarding the sustained use of Computer Algebra Systems (CAS) in university instruction. CAS use among individual professors and its sustained use in university departments are timely topics within a technological era. Instructor beliefs regarding the nature of mathematics learning, required curriculum/assessment changes, the use of technology in one's own research, and the availability of resources are among the complex set of factors that affect the degree to which technology is implemented within undergraduate university mathematics courses. A Canadian national survey of over 300 participating mathematicians indicated that many professors are using CAS in their instructional practice, and that the greatest factor influencing the use of CAS in one's post-secondary mathematics teaching is the use of CAS in one's own research. While individual adoption of CAS and other instructional technologies may be popular among technology enthusiasts, the long-term adoption of these technologies across an entire mathematics department faculty is much more difficult. Findings from two departmental case studies in the United Kingdom and Canada indicate that a sustained implementation at the departmental level requires a unique combination of key factors and strategies such as: a dedicated core group led by a committed advocate in a position of influence/power (e.g., Head/Chair); a strong and shared incentive for change; strategic hiring processes; an administration which supports creative pedagogical reform and well-considered risk-taking; and a continuous and determined revisiting of the original vision and purpose. Significant challenges to implementation are also discussed.

1. Research Context and Overview

Buteau, Jarvis, and Lavicza were invited to participate in the International Commission on Mathematical Instruction (ICMI) Study # 17 entitled, *Digital technologies and mathematics teaching and learning: Rethinking the terrain*, which was held in Hanoi, Vietnam in December 2006. It was here that they first met, and in subsequent correspondence applied for, and were granted national research funding (Social Sciences and Humanities Research Council of Canada International Opportunity Development and Project Grants, 2007/2008) to complete a mixed-methods research study focusing on Computer Algebra System usage by mathematics instructors at the university level, and on departmental change in light of technology implementation in curriculum and assessment.

> Although the computer hardware and software options have been present for decades, we have still not seen a major shift in pedagogy within our education systems such as was widely predicted. . . . We need to dedicate perhaps 10% of our individual energy and working lives to the exploration of new ways of teaching—of reconceptualizing how it is that we teach and students learn mathematics at all levels.

It was with these powerful and challenging opening remarks that Dr. Seymour Papert addressed the international group of researchers attending the International Commission on Mathematical Instruction (ICMI) Study # 17 in Hanoi, Vietnam in December 2006. Dr. Papert is widely known for his research and innovations in the areas of Artificial Intelligence, the development of Logo for early childhood instruction, his work with Piaget in the 1960s around constructivism, his teaching and research at MIT, and his most recent initiative—the "$100-dollar laptop." The entire ICMI study conference, featuring working groups, lectures, and computer lab sessions, focused on technology usage within mathematics education at all levels. Our SSHRC research study was, at least in part, a shared response of Jarvis, Lavicza, and Buteau to Papert's challenge to rethink the status quo mathematics pedagogy found in universities. We wished to facilitate, via an international mixed-methods research study, a positive move forward with regard to digital technologies in the university mathematics curriculum.

A growing number of international studies have shown that Computer Algebra Systems (CAS-based) instruction has the potential to positively affect the teaching and learning of mathematics at various levels of the education system, even though this has not been widely realized in schools and institutions [1, 4, 15, 18, 23, 24, 26]. In contrast to the large body of research focusing on technology usage that exists at the secondary school level, there is a definite lack of parallel research at the post-secondary level. Furthermore, although substantial research has been conducted surrounding the area of professional development for teachers [9, 10, 11], relatively little has focused specifically on professional development for mathematics educators [3, 21, 25], especially with regard to technology usage at the university level. As noted by Lawless and Pellegrino [20], large meta-analysis studies on professional development have indicated that there are several key components necessary for success, one of those being the "access to new technologies":

> A number of organizations and researchers have conducted elaborate reviews of the literature and evaluations in this area. . . . This knowledge base has consistently indicated that high-quality professional development activities are longer in duration (contact hours plus follow-up), provide access to new technologies for teaching and learning, actively engage teachers in meaningful and relevant activities for their individual contexts, promote peer collaboration and community building, and have a clearly articulated and a common vision for student achievement. (p. 579)

Since relatively little research had been done relating to the support of mathematicians using CAS-based instruction at the university level, this became our primary focus. Our research program aimed at both documenting university teaching practices involving technology, and formulating recommendations for individual and departmental change. We also wanted to contribute to increasing the amount of attention paid to tertiary mathematics teaching, from a research perspective, and to seeing more related articles published which would elaborate on specific issues and strategies for systemic integration of technology in university mathematics curriculum. In what follows, we summarize our research by bringing together the main parts of our research study. We end by drawing some preliminary connections between the findings that have resulted from each of these parts.

2. Research Methodology

Our research study was structured around three main research components, as well as the organization of two 1-day workshops for post-secondary mathematics instructors. Three data-gathering research initiatives were implemented in 2008-2010: (i) a preliminary literature review (326 papers) was completed, followed by a more comprehensive literature review analysis; (ii) a nation-wide, bilingual, online survey of Canadian mathematicians in post-secondary institutions (universities/CÉGEPs[1]) regarding CAS use in research and teaching to compare with international trends [19]; and, (iii) two in-depth case studies (Canada, United Kingdom) of university departments in which the use of technology in mathematics instruction appeared to have been heavily adopted by a mathematics department and sustained over time.

2.1 Literature Review

In the summer of 2008, we began a pilot study focusing on 326 contributions dealing with CAS use in secondary/tertiary education and technology use in tertiary education. These papers were drawn from two well-regarded journals, namely the *International Journal for Computers in Mathematical Learning* (issues since its beginning in 1996) and the *Educational Studies in Mathematics* (since 1990). We also selected proceedings from two technology-focused conferences, namely the *Computer Algebra in Mathematics Education* (since its first meeting in 1999) and the *International Conference on Technology in Collegiate Mathematics* (since 1994 with first electronic proceedings). A sub-corpus of 204 papers that in whole or in part explicitly discuss CAS use at the post-secondary level was then identified to further focus the analysis.

Based on the Systematic Research Synthesis methodology developed by the EPPI Centre at the University of London (EPPI-Centre, 2007) and guided by the Lagrange et al. [19] theoretical framework, we progressively developed our own framework. More precisely, while the descriptive themes found within the Lagrange template [19] were helpful, we began to notice that some of them would need to be adapted and several other theme columns would be beneficial at this stage of the template development. We modified subthemes of some themes, such as "technology used" and "mathematical fields," and we also added the following themes: "computer/calculator", "integration scope", "instructional purposes", "course level", "examples of CAS use", and "implementation issues". For the theme involving instructional use of CAS, we used as subthemes the eight purposes identified by Lavicza [19] in his international (US, UK, Hungary) comparative survey (p. 164). After the review was completed, we decided to add the theme of "potential benefits of CAS."

[1] CÉGEP is an acronym for *Collège d'enseignement général et professionnel*, and refers to the public post-secondary education collegiate institutions exclusive to the education system in the province of Quebec in Canada.

An important point to note here is that in contrast to the Lagrange study where a significant proportion (38%) of papers were those describing educational research results, our selection of papers revealed a majority that focused on practitioner innovations with very few (10%) involving educational research (see Table 1). It became clear that we could not set aside the 90% of contributions from practitioners if we wanted to fairly report on CAS-based technology integration in post-secondary mathematics education, and so we have reported on the content of the 90% papers in several existing publications [7, 8, 22].

Table 1: *types of contributions found in the literature review [8]*

Practitioner Reports	Presentation of Examples	46%
	Examples with practitioner reflections	20%
	Classroom Study	6%
	Classroom Survey	3%
	Examinations of a specific issue	3%
	Abstract only	11%
Education Research Papers		10%
Editorial Journal Letters		1%

2.2 Canadian Mathematician Survey

The Canadian survey study was designed to obtain a preliminary picture of CAS integration in Canadian post-secondary mathematics instruction. Overall the study addressed the following three issues: (i) the extent of CAS use in post-secondary institutions; (ii) views of mathematicians on the role of CAS in mathematics literacy and curriculum; and, (iii) factors influencing the integration of CAS into mathematics teaching and learning at the post-secondary level. The study was also designed to compare the results with those of a similar international survey study conducted in the United States, United Kingdom, and Hungary [19]. Lavicza became part of the research team for the Canadian study and as such, this research can be viewed as a form of direct extension of his international survey study. We therefore used a similar methodology (participant recruitment, questionnaire, and analytical tools) as in the Lavicza [19] research, but with slight modifications to the survey (e.g., Canadian context question modifications, French translation for Francophone participants) where deemed appropriate.

For the Canadian survey study, we slightly adapted the methodology to take into account the Canadian reality. For example, this meant that the survey questionnaire first needed to be prepared in both official languages, English and French. Although there were several issues that warranted further investigation and/or changes in wording to the survey, we decided to align the Canadian survey items very closely with the Lavicza [19] survey for the sake of direct comparisons. Furthermore, to increase participation we did not want to extend the length of the time (approximately 20 minutes) needed to complete the survey. A decision was therefore made to insert only two additional questions relating to the use of broader technologies in research and teaching.

The analysis of the data was guided by Lavicza's similar analysis of his international survey data. Descriptive analysis was performed on all the Canadian data, as well as inferential analysis for comparing results from CAS users and non-CAS users. In addition, an exploratory factor analysis was conducted to identify characteristics of mathematicians who indicated that they do integrate CAS in their teaching. For the comparison with international trends, this was done in two ways: (i) treating Canada as an additional (i.e., fourth) participating country; and, (ii) contrasting Canadian results directly with the combined international results. The written responses were analyzed with the same method as for the international study.

Nearly 2000 personalized emails were sent to all mathematicians listed in official websites of 60 departments of mathematics in Canadian universities, plus one set of subsequent personalized email reminders. For CÉGEP mathematics instructors, we used a mailing list (54 of 56 public and private CÉGEPS, which included 170 instructors) through their main professional association, *Association de mathématique du Québec*. Personalized emails were all bilingual. Both versions (French and English) of the on-line survey were located on Free Online Survey system. Eventually, 302 mathematicians responded to the online survey, 223 to the English version and 79 to the French version, with a total response rate of about 14.5%.

2.3 Comparative Case Study

In preparation for the case studies, we first consulted widely in the fields of mathematics and mathematics education at conferences and at our own respective universities and regions to ascertain what types of technology use were happening in different parts of the world. In this part of the research, we didn't restrict our focus to CAS-based technology only, but rather considered any digital technologies being used by mathematics departments. Over time, as a research team, we narrowed a list down to 5-10 possible institutions in North America and Europe where we could definitely see signs of a unique, sustained technology-saturated undergraduate mathematics program. In the end, we settled on two such institutions, one in Canada and one in the United Kingdom for the comparative case study, due to their long-term technology-intense programs, and also due to the relative accessibility (i.e., researchers employed in these countries). The universities in which these two mathematics departments were located were then contacted, and agreements were eventually drafted and signed at each location to allow for a department-wide research event to take place in both venues.

Two case studies were then carried out. Interviews were conducted with key individuals (administrators and faculty) at both the UK and Canadian sites. The interview questions were semi-structured (i.e., open-ended in nature) and designed according to case study standards. Participants were thereby encouraged to communicate their individual perceptions relating to their departmental structures, leadership, resources, and significant changes over time—particularly those involving the re-crafting of the undergraduate mathematics programs with a comprehensive use of digital technology. Artefacts such as course schedules, sample assignments, marketing brochures, webpages, and meeting minutes were also collected for review. These items were helpful in ensuring the overall accuracy of the analysis of participants' statements, and also provided further information about the two undergraduate programs. Interviews were transcribed using Sony Digital Voice Editor and Express Scribe and checked for accuracy. Transcripts were then analyzed using Atlas.ti software for qualitative research and Thematic Analysis methods, with the emergent themes from the data coding process, discussed in further detail below, used in the crafting of the case study report.

The United Kingdom Maths Group (i.e., department) was situated within a large-sized university (i.e., student population of approximately 34 000) in north-central England. During several visits to the university site in early 2009, interviews were scheduled with nine individuals including mathematics professors and the department Head/Chair. Similar supporting artefacts were also collected at the UK site. The UK interviews were conducted by the same two researchers, and the interview schedule of questions was the same for each of the participants.

The Canadian mathematics department was situated within a medium-sized university (i.e., student population of approximately 17 000) in Ontario, Canada. Over a number of days in late 2008, interviews were scheduled with 19 individuals at this university including mathematics professors, the department Chair, and a retired faculty member who had been instrumental in implementing the program. One further interview with a senior administrator took place by telephone several days following the original interviews. Similar type artefacts were also collected at the second site. Two researchers conducted the interviews together, taking turns asking questions from the same prepared interview schedule.

3. Research Findings

Computer Algebra Systems (CAS) are increasingly being used in university-level mathematics education [19]. However, little is known about the extent of CAS use and the factors influencing its integration into university curricula. Pre-university level studies suggest that beyond the availability of technology, teachers' conceptions and cultural elements are key factors in technology integration into mathematics teaching and learning [16, 2]. The Lavicza research study [19] featured an on-line survey of 1100 mathematicians as well as interviews with 22 mathematicians in three countries, namely, Hungary, United Kingdom, and United States, which examined mathematicians' conceptions of CAS and its pedagogic uses. Findings showed some similarities, but numerous differences, between university- and school-level research findings. Building on Lavicza's international doctoral study, our 3-year, mixed-methods research study examined individual and systemic CAS implementation and sustained usage in undergraduate mathematics teaching.

3.1 Literature Review Findings

The literature review was done in two phases. In a preliminary pilot study, 326 educational research and practitioner report contributions were reviewed in order to refine the theoretical framework proposed by Lagrange et al. [17] to better address tertiary education.

Several themes emerged from the review focused on practitioner reports: diverse uses of CAS, benefits to student learning, issues of integration and mathematics learning, common and innovative usage of CAS, and integration scope in university curricula. These findings are discussed at length in Buteau, Marshall, Jarvis, and Lavicza [2010]. Our analysis suggests, in particular, that, perhaps contrary to popular belief, CAS integration in tertiary mathematics teaching occurs most frequently in courses for mathematics majors as opposed to service courses designed for non-math majors (see Table 2). The most commonly examined issue encountered in the literature was that of assessment [7]. Furthermore, it was noted that overall, there appears to be less concern among mathematics instructors about technical and financial issues than there is for pedagogical issues [7].

Table 2: *student types for CAS integration/instruction reported in the literature review [8]*

Mathematics Majors	First Year Maths Majors	32%
	Upper Year Maths Majors	24%
Engineering and Science Majors		4%
Teacher Education Majors		2%
Other Program Majors (e.g., Business Majors, Social Science Majors)		2%
Unstated Undergraduate Program		37%

The main uses of CAS in instruction reported were: (i) for experimentation and exploration, (ii) visualization, and, (iii) real and complex problems. It was also observed that in many instances instructors would refer to common uses of CAS described in their work as being "new," underlying the need for better communication and shared teaching material/resources between mathematicians. Most reported use of CAS technology reflected integration by a single instructor, whereas there was very little indication of program-wide CAS use (see Table 3). Most of these results corroborate with those of Lavicza's [19] international survey study [22].

Table 3: *CAS technology implementation scope regarding tertiary mathematics instruction [8]*

One Course	67%
Series of Courses (Calculus, Linear Algebra, etc.)	16%
Grouping of courses (First-year courses, etc.)	11%
Program-wide implementation	6%

3.2 Canadian Mathematician Survey Findings

The survey was completed in 2009 as an extension of Lavicza's doctoral study [19] in which he surveyed over 4500 mathematicians from Hungary, the United Kingdom, and the United States. In his work, Lavicza had debunked several common myths such as the notion that few professors use CAS technology in their teaching, or that only younger professors adopt technology for instruction. With a remarkably high response rate (1100 respondents), many mathematicians in the Lavicza study [19] shared their own beliefs and practices concerning CAS use in mathematics instruction, providing dozens of pages of anecdotal experiences and commentary. He also demonstrated, using Structural Equation Modeling analysis, that the strongest predictor for the use of CAS in one's teaching was clearly the use of CAS in one's research.

As part of our study in Spring 2009, we conducted a similar national online survey (302 respondents) regarding the use of Computer Algebra Systems (CAS) in post-secondary mathematics teaching [6]. Of the estimated 2300 mathematicians working at Canadian universities and CÉGEPs (Québec), 302 took part in our 32-item survey, which featured both Likert-scale ranking and open-response items. A larger majority of participants (69%) reported using CAS in their teaching, with an even larger proportion (81%) of participants indicating using it in their research. A variety of different ways of using CAS in teaching were reported in the study (Figure 1).

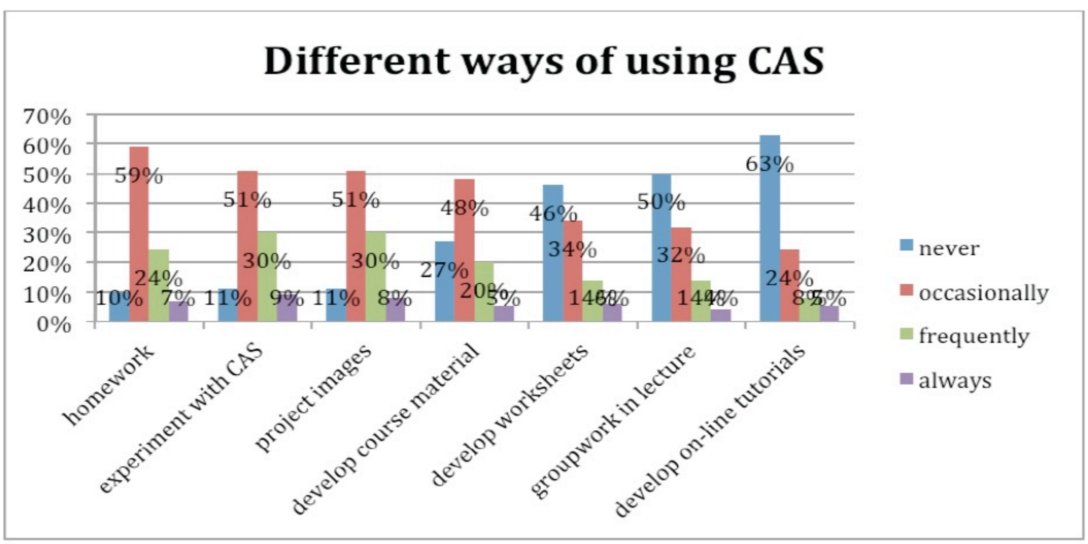

Figure 1: *the different uses of CAS in instruction*

The lack of enthusiasm by colleagues towards using CAS in mathematics classes, the time required for developing CAS-related teaching material, and the tight class time making it difficult to 'add' CAS-related activities were identified by participants as being the most significant factors that may hinder the integration of CAS into one's teaching. Non-CAS user mathematicians mostly agreed that the reason that they did not integrate technology is related to time: time to redesign a course, to develop teaching material, or for students to learn to use the software. In terms of assessment, only 22% of all CAS-user mathematician participants integrated CAS in final exams. Our analysis revealed that mathematicians did not agree whether CAS use affects, or does not affect, the mathematics that has to be learned by students in post-secondary institutions.

Our Canadian survey also contained two questions relating to the use of broader technology (CAS, programming, SAS, dynamic geometry software, discrete mathematics software, simulation software) in teaching and research. Except for computer programming, all different forms of technology (i.e., used specifically for doing mathematics, and not including communication technology) used by Canadian mathematicians in mathematics teaching and research work seem to be integrated relatively to the same extent.

Some significant differences in English and French responses were also noted and discussed during the Canadian workshop research presentations [5], as were interesting findings pertaining to mathematicians' beliefs regarding technology use, including actual assessment practices. A full report of the survey findings can be found in [6].

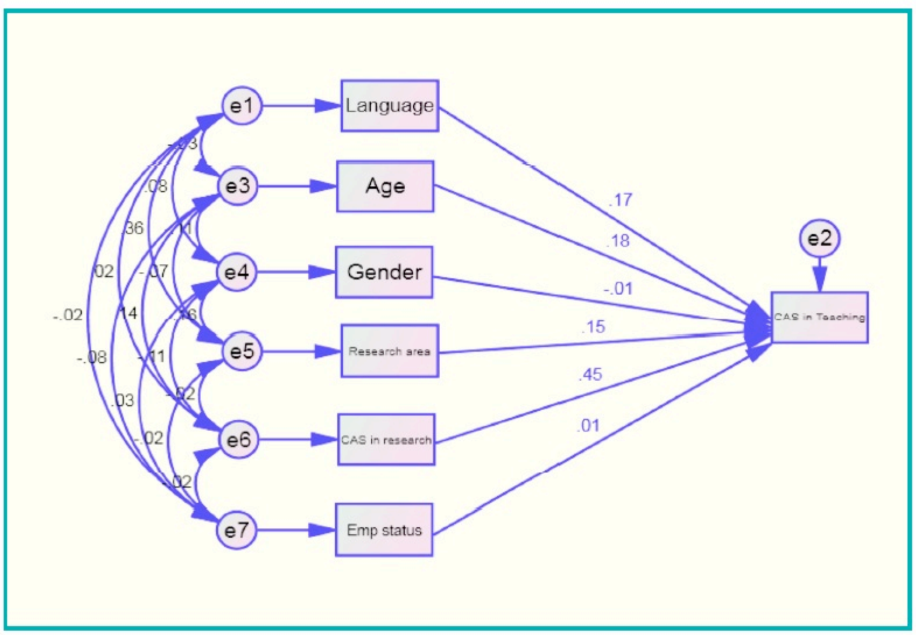

Figure 2: *diagram summarizing the exploratory factor analysis (EFA) regarding the use of CAS in teaching by use of a Structural Equation Modeling (SEM) regression model (Chi-square = 0.000; df = 0; p = 0; CFI = 1.000; RMSEA = 0.127)*

Figure 2 shows the observed variables affecting CAS use in teaching: language (French, English), age, gender, research area, integration of CAS in research, and, employment status. Lavicza's early survey results [19] highlighting the strong relationship between *CAS use in one's research and CAS use in one's teaching* was again found to exist in the analysis of the Canadian survey data, with an even stronger correlation.

3.3 Comparative Case Study Findings

Two case studies were conducted in universities where there appeared to be significant, long-term adoption of CAS (or more broadly speaking, technology) in the teaching of mathematics within an entire department. Interviews were conducted with key individuals (instructors, administrators, heads/chairs) at both the Canadian and UK institutions. Emergent themes from data coding included the following: History of Math Program; Instructor Background; Structure of the Program; Rationale for Program Creation; Obstacles/Challenges; Recruitment/Marketing; Learning Community; Math Concept Learning; Technology Used; Assessment; Student Projects; and Sustained Departmental Shift.

United Kingdom BSc Mathematics Degree Program

Note: To obtain a BSc, students must accumulate 360 credits, 120 at each of levels 4, 5 and 6.

Year 1 (Level 4): All modules 20 credits, except where stated

Semester 1	Semester 2
Mathematical Modeling; Mathematical Technology; Number and Structure; Mathematical Methods; and, Statistics and Probability	
Elective 1: Maths Workshop 2 (10); History of Maths (10); Exploring the Universe (10); or, a Modern Language course	**Elective 2:** Basic Computer Programming (10); Dynamic Geometry (10); or, a Modern Language course

Year 2 (Level 5): All modules 20 credits

Semester 1	Semester 2
Modelling 2; Linear and Discrete Mathematics; Dynamical Systems and Fourier Analysis; and, Statistical Methods	
Elective (choose one): Analytical Research Methods; Business Mathematics; Optimisation Methods; Programming for Excel and the Web; C and C++ Programming; or, a Modern Language course	

Year 3 (Industrial Placement)

Year 4 (Level 6): All modules 20 credits, except where stated

Semester 1	Semester 2
Project (30); Advanced Mathematical Case Studies	
Professional Development (10)	
Elective (choose three): Digital Signal Processing; Modelling with Partial Differential Equations; Fluid Flow; Tensors; Control Theory; Multivariate Statistics and Data Mining; Statistics for Business; Scheduling Applications; Advanced Web Programming and Parallel Computational Mathematics; or, a Modern Language course	

Canadian BSc Combined [TECH] and Math Specialization Honours Degree Program

Year 1

Term 1	Term 2
Linear Algebra I; Calculus I	[TECH] I; Calculus II

Year 2

Required by all math students: [TECH] II (full year); Linear Algebra II; Calculus III; Ordinary Differential Equations; Probability; Statistics I

Pure Math Stream: Intro to Analysis; Abstract Linear Algebra; Discrete Optimization
Applied Math Stream: Intro to Analysis; Discrete Optimization
Statistics Stream: Experimental Design; Euclidean and Non-Euclidian Geometry I; Intro to Financial Math
Education Stream: Intro to Combinatorics; Euclidean and Non-Euclidean Geometry I; Great Moments in Mathematics I; Mathematics and Music

Year 3

Pure Math Stream: Intro to Combinatorics; Real Analysis; Complex Analysis; Group Theory; Abstract Algebra; Game Theory; Intro to Topology
Applied Math Stream: Complex Analysis; Advanced Differential Equations; Partial Differential Equations; Group Theory; Numerical Methods; Continuous Optimization; Theory of Financial Math
Statistics Stream: Experimental Design; Regression Analysis; Statistics II; Applied Multivariate Statistics; Advanced Differential Equations; Partial Differential Equations
Education Stream: Group Theory; Euclidean and Non-Euclidean Geometry II; Math at the Junior/Intermediate/Senior Level; Great Moments in Mathematics II; Abstract Algebra

Year 4

Pure Math Stream: Advanced Real Analysis; Topics in Groups; Combinatorics; Topics in Number Theory and Cryptography; Topics in Topology and Dynamical Systems; Topics in Rings and Modules; Honours Project
Applied Math Stream: Intro to Wavelets; Topics in Differential Equations; Solutions and Integrability of Nonlinear Evolution Equations; Topics in Stochastic Processes and Models; Topics in Advanced Statistics; Honours Project
Statistics Stream: Sampling Theory; Nonparametric Statistics; Topics in Stochastic Processes and Models; Topics in Advanced Statistics; Honours Project
Education Stream: Combinatorics; Topics in Groups; Topics in Rings and Modules; Advanced Mathematical Structures; Honours Project

Figure 3: *program course requirements for the UK and Canadian case study sites*

What has emerged from the data analysis of the interview transcripts and artefact analysis from the two sites is that sustained long-term shifts in departmental technology use in mathematics instruction appear to require the following components for successful implementation: a key proponent in a position of influence/power (e.g., Head/Chair, often supported by one or more energetic, technology-savvy radicals and well-seasoned politicos); a strong and shared incentive for change; strategic hiring practices, where possible; an administration which supports creative pedagogical reform and well-considered risk-taking; and, a continuous and determined revisiting of the original program vision. We have also learned that despite the best and tireless efforts of the core group leadership, the growth in faculty numbers and program scope within a healthy department, the implications of academic freedom on teaching practices, and the inevitable retirement of key players, makes the process of program maintenance and long-term coherence much more challenging at the university level. A full report of the comparative case study analysis has been recently published [14].

4. Workshops at National Mathematics Research Institutes

In October 2010 we hosted two single-day events entitled, *Workshop on Technology Integration in Teaching Undergraduate Mathematics Students,* first in French at the *Centre de recherche mathématique* (CRM) in Montreal, and then in English at the *Fields Institute for Research in Mathematical Sciences.* These two workshops aimed at fostering discussion among mathematicians and mathematics educators about technology integration in university education. These workshops related directly to our research program insofar as they were designed to encourage conversations and sharing regarding technology use among Canadian mathematicians. Note that the PIMS mathematical research institute in British Columbia declined our workshop proposal, and as such, workshops were unfortunately only hosted in the eastern part of Canada.

Three rather unique aspects of our workshops were the following: (i) the two mathematics education workshops took place in mathematics research institutes; (ii) the CRM workshop was apparently the first "mathematics education" event ever to take place at the center since its inception in 1968; and, (iii) the keynote speaker and discussion group leaders all participated in both events, speaking/hosting in French in Montréal, and then in English in Toronto. Furthermore, we published full reports of these workshops in the Field's Institute's official newsletter, *Fields Notes* [5], and in the Canadian Mathematics Education Study Group's (CMESG) *Newsletter* [13].

The programs were comprised of a keynote address by Bernard Hodgson (*Postsecondary mathematics education and technology: Some personal views from a mathematician's perspective*), former Secretary for the International Commission on Mathematical Instruction (ICMI); a presentation of our Canadian survey results; and, three discussion groups lead by invited scholars on important topics (*Rethinking the undergraduate mathematics curriculum: What role does technology play?* with Whiteley, Saliola, & Heffernan; *Integrating an additional Junior course in programming and/or CAS use: Issues and impact on curriculum*, with St-Aubin, Van Rensburg, & Delisle; and, *Assessment in mathematics courses integrating technology: Technical/pedagogical challenges and curricular considerations*, with Caron & Ben-El-Mechaiekh). [5]

5. Concluding Thoughts

In this paper, we have summarized our SSHRC-funded research by bringing together all parts of our research study. Our research has lead to conference papers presented at international conferences (Canada/USA/UK/France/Korea), the hosting and reporting of a Canadian Mathematical Society (CMS) Winter Meeting Technology Session [12], writing of several journal articles, and, the hosting of two workshops at national mathematics research institutes (CRM, Montreal; Fields Institute, Toronto). Further, the research team designed a comprehensive website which features the following sub-pages: Introduction, Literature Review, Canadian Survey, Case Studies, Workshops, Presentations, Publications, and Resources (see project website: http://www.nipissingu.ca/casresearch/index.htm)

Findings from the literature review, Canadian survey, and comparative case study, as well as discussions at the national workshops all indicate that while Computer Algebra System (CAS) tools/software and other technologies are frequently used by enthusiastic mathematics instructors at the post-secondary level, system-wide adoption of such practices across mathematics departments remains rare and represents a complex and lengthy undertaking. The use of CAS in one's research was clearly shown to be the most significant factor affecting the use of CAS in one's teaching, a Canadian finding which reinforces Lavicza's international results [19], and one which is perhaps not surprising given the relatively short amount of time required by researchers to implement the software in classes/labs, who are already proficient with CAS technology and excited about using it with their students. This coincides also with the realities observed in the two mathematics departments in the case studies.

Furthermore, there appears to be a lack of a centralized system for sharing rich CAS-based curriculum and assessment tools (i.e., relatively little communication between faculty members and universities). One of our research goals was to help promote this type of communication among Canadian mathematics professors. Our workshops and website attempted to address this problem, and have perhaps went some distance in doing so, but we are hopeful that other mathematician/researchers will develop excellent resources along with an accessible system for sharing these among colleagues. Findings from the literature review, national survey, and international case studies all point to the need of further research in the area of CAS use and sustainability in undergraduate mathematics teaching. Insights from the various research components, as reported in more detail in other publications noted herein, provide individual mathematics instructors and those in positions of departmental leadership with positive change strategies regarding the implementation of CAS and other technologies in undergraduate mathematics teaching. We contend that such instructional initiatives, although difficult and time-consuming, are timely and worthwhile endeavours, which can lead to deeper understanding and more engaging mathematics learning experiences for 21st century, post-secondary learners.

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