Computers & Education 58 (2012) 423–434

Contents lists available at SciVerse ScienceDirect

Computers & Education

journal homepage: www.elsevier.com/locate/compedu

Do mathematicians integrate computer algebra systems in university teaching? Comparing a literature review to an international survey study

Neil Marshall [a,*], Chantal Buteau [a], Daniel H. Jarvis [b], Zsolt Lavicza [c]

[a] Department of Mathematics, Brock University, 500 Glenridge Avenue, St.Catharines, Ontario, Canada L2S 3A1
[b] Schulich School of Education, Nipissing University, North Bay, Ontario, Canada P8B 1L7
[c] Faculty of Education, University of Cambridge, 184 Hills Road, Cambridge CB2 8PQ, UK

A R T I C L E I N F O

Article history:
Received 10 February 2011
Received in revised form
13 July 2011
Accepted 16 August 2011

Keywords:
Applications in subject areas
Post-secondary education
Human–computer interface
Improving classroom teaching

A B S T R A C T

We present a comparative study of a literature review of 326 selected contributions (Buteau, Marshall, Jarvis & Lavicza, 2010) to an international (US, UK, Hungary) survey of mathematicians (Lavicza, 2008) regarding the use of Computer Algebra Systems (CAS) in post-secondary mathematics education. The comparison results are organized with respect to four emerging themes: Issues in CAS integration and mathematical learning, the notion of mathematical literacy, diverse uses of CAS by practitioners, and, potential benefits of CAS integration. Our analysis suggests that the results of the literature review strongly support the findings and conclusions of Lavicza's international survey. We contend that Lavicza's concluding statement about the need to more holistically examine technology integration in post-secondary mathematics departments was significantly realized through both of the compared studies.

1. Introduction

With the advent of the digital era, emerging technologies such as computer algebra systems (CAS) brought with them the potential to shape new directions in the teaching and learning of mathematics. In spite of the fact that a growing number of international studies have demonstrated that CAS-based instruction has the potential to positively influence the teaching and learning of mathematics at various levels of the education system, CAS has not been widely implemented in schools and higher education institutions (see Artigue, 2002; Lavicza, 2006; Pierce & Stacey, 2004).

Also while there exists a large body of research focussing on technology use at the secondary school level, there is clearly a paucity of parallel research in post-secondary mathematics education. This was indeed demonstrated in the 2006 International Commission on Mathematics Instruction (ICMI) Study 17 (Hoyles & Lagrange, 2010; Son, Sinclair, Lagrange, & Hoyles, 2006), entitled ' Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain', at which only very few contributions addressed post-secondary instruction. It is similarly portrayed in the comprehensive literature review study by Lagrange, Artigue, Laborde, and Trouche (2003), which dealt with technology and mathematics education research and innovation and in which only a few of the reviewed contributions addressed post-secondary education.

One recent international study of interest is that of Lavicza's (2008) in which he focused on mathematicians' use of Computer Algebra System (CAS) in their post-secondary teaching and research. Featuring a comparative analysis of academic mathematicians' conceptions and professional use of CAS in post-secondary mathematics, the study attempted to address perceived deficiencies in educational research about CAS use at the post-secondary level. Using stratified random sampling, Lavicza surveyed mathematicians from Hungary, the United Kingdom and the United States, regarding their conceptions and teaching practices with CAS. Over 1100 mathematicians participated from these three countries, some of whom integrated technology into their teaching, and some of whom used only traditional instruction.

Another study which also examined CAS use in post-secondary mathematics education was the literature review by Buteau, Marshall, Jarvis, and Lavicza (2010). Though it was a pilot study, mainly aimed at developing a theoretical framework for a larger in-depth review,

* Corresponding author. Tel.: +1 905 688 5550x3300; fax: +1 905 378 5713.
E-mail address: nm00ay@brocku.ca (N. Marshall).

0360-1315/$ – see front matter
doi:10.1016/j.compedu.2011.08.020

questions emerged regarding these two studies, since they examined the same topic using different sources. Did the results of the literature review corroborate with those of the international survey? Were there differences in what mathematicians were writing in peer-reviewed journals and conference proceedings and how they responded to an individual questionnaire? Furthermore, could the results and conclusions of the literature review contribute to the major conclusions of the international survey, and vice versa? It is in the context of addressing these questions that we propose to compare the literature review with the international survey. We submit that an examination of both studies, emphasizing similar, complementary, and contrasting results may indeed prove informative.

In this paper we present details of our comparative study of the literature review (Buteau et al., 2010) to the international survey study (Lavicza, 2008). In Section 2, we discuss the methodology. We present our findings in Section 3 according to two levels of comparison. From the first level, we illustrate our findings according to the following four emerging themes: Issues in CAS integration and mathematical learning, the notion of post-secondary mathematical literacy, diverse uses of CAS by practitioners, and, benefits to CAS integration. From the second comparison level, we discuss the potential contributions of the literature review to the international survey conclusions, and vice versa. We conclude in Section 4 with some final remarks summarizing further results of both studies, and discuss future directions of our research.

2. Methodology

Buteau et al. (2010) examined, for their small-scale literature review, 326 contributions from two well-regarded journals, *International Journal for Computers in Mathematical Learning* (IJCML), and *Educational Studies in Mathematics* (ESM), and from two technology-focused conference proceedings, *Computer Algebra in Mathematics Education* (CAME) and the *International Conference on Technology in Collegiate Mathematics* (ICTCM). A sub-corpus of 204 papers that dealt with post-secondary mathematics instruction with CAS use had been selected for the analysis. For our proposed comparative study, the literature review data consisted of our completed template[1] of detailed notes on the literature review corpus used to conduct the literature review, archived paper corpus, and our paper (Buteau et al., 2010) which presents detailed results of the study. Since the template included educational research papers, we excluded those papers for a more accurate comparison with Lavicza's study about practitioners. The literature review was comprised of contributions from 1990 to 2008. The results from the literature review for our comparative study consisted of: (i) basic statistical descriptions of the themes that had emerged throughout the study, and (ii) exemplary quotations from the literature review which dealt with major themes.

Lavicza's international survey consisted of two distinct phases. The first phase involved a qualitative investigation using a Grounded Theory approach in which individual practitioners were interviewed and their mathematics classes attended by the researcher in order to identify important issues in relation to CAS integration. This was followed by a second, quantitative study involving an online, international survey of mathematicians and which examined, with a larger population, the issues emerging from Phase I. The international survey data for our comparative study strictly consisted of Lavicza's Ph.D. thesis. The international survey focused on mathematicians' use of CAS in post-secondary teaching. Two chapters of the Ph.D. thesis were initially selected for possible overlap with our literature review: *Current use of CAS at universities,* and *Mathematicians' conceptions of CAS use in mathematics teaching*[2]. These two chapters elaborate upon Lavicza's study Phase II, with a particular focus on how mathematicians use CAS in their teaching and in their research, and their views pertaining to CAS and its relationship to mathematics. Other chapters elaborate on the following topics: Lavicza's study Phase I the development of the questionnaire for the international survey, characteristics of the survey participants, influences on CAS integration at post-secondary institutions, and the discussion and implication of results.

Using the two chapters as a basis, only some sections, *Platforms of CAS use*, *Purposes of CAS use*, *Assessment*, *CAS use by subject area,* and *Mathematicians' conceptions*[3], were selected for our comparative study since the literature review lacked data for a proper comparison of the other sections. The latter[4] were thus omitted for this reason or due to a lack of relevance to this study.

Results from the selected sections for our comparative study consisted of: (i) a basic statistical description of the survey questions (Likert style), and (ii) Lavicza's analysis of (i) that also made use of mathematicians' quotes from Lavicza's study Phase I and from the open survey questions. As a result, that which professors were writing in response to a survey of mathematical practices and CAS use could be compared to that which mathematicians were writing about CAS (in educational contexts) in journals and conference proceedings.

The data analysis proceeded according to two comparison levels. The first level consisted of direct result comparisons as follows: For each result documented by Lavicza, the literature review data was examined to see if there were common, complementary, or contradicting results, with meticulous notes being created accordingly. It was determined from these notes that direct comparison of the quantitative data was in most cases not possible since Lavicza's statistics were often too specific to be compared to those of the literature review. We were able to compare some basic statistics such as platform of CAS use, major issues, and purposes of CAS use. As for the comparison of qualitative results from both studies, in reviewing the comparison notes, we identified themes from both studies that had comparable data. These notes were then re-organized with respect to these four emerging common themes: two from the literature review (Potential Benefits and Issues), one from the international study (Mathematical Literacy) and one from both studies (Purposes of CAS Use, which was taken in the literature review directly from the international survey). This approach was chosen for readability and organization. Lavicza's Ph.D thesis presentation structure was comprehensive and, as such, was not directly suitable for our comparison purposes. For example, issues were intermixed with benefits and spread throughout both chapters used for our comparison study.

As opposed to the other themes, mathematical literacy had not been initially identified in the literature review. However, as previously mentioned, the main purpose of this literature review was to develop the framework for a subsequent, in-depth literature review, and as such, a column in the template had been added for inserting comprehensive notes of interest that could be used as the template was evolving during the literature review. After taking a close look at Lavicza's results, we compared them with data from the literature review

[1] Filled excel file with extensive categories and elaborated coding system; see Buteau et al. (2010) for details.
[2] Chapters 9 and 10, respectively.
[3] Sections 9.2, 9.4, 9.6, and 9.8, respectively.
[4] Sections 9.3, 9.5, 9.7, 9.9, 9.10, 10.5 and 10.6.

(from the notes column in the template) and noticed that we did have substantial data for further analysis. We thereafter carefully reviewed the column notes, sometimes going back to the corpus, in order to bring this theme into the comparison (as a result, the mathematical literacy category has been inserted into the template for our ongoing comprehensive literature review).

The second comparison level consisted of identifying potential contributions of the literature review to the international survey conclusions, and vice versa. For this purpose, Venn diagram tables were created using the main results and conclusions of both studies. The intersection contained all results clustered in the four themes from the first comparison level; the strictly literature review (sLR) part contained the results from level 1 that were not regrouped in the four themes, and their related conclusions; and the strictly international survey (sIS) part contained the major results of the whole study, beyond those results found in comparison level 1 with its five concluding implications[5]. Contributing factors between sLR and sIS were then highlighted with respect to the concluding implications. Reverse contribution relations in the Venn diagram were either not possible or nor of interest, due to the much smaller research scope of the literature review study. Instead, contributing factors from sIS, using the main study results, to areas of potential bias in the literature review were drawn.

3. Results and discussion

We first compare the results of the two studies in Section 3.1. In Section 3.2 we discuss the conclusions of the international survey and how the literature review may further them as well as how the international survey may contribute to limitations and findings of the literature review.

3.1. Comparing the results of both studies

In comparing the results of the literature review to Lavicza's international survey, we examine four major themes: issues of CAS-technology integration and mathematics learning with the use of CAS, mathematical literacy, benefits of CAS integration, and, diverse use of technology. In what follows, we present highlights from within these four emergent themes.

3.1.1. Issues in CAS integration and mathematics learning

The international survey (Lavicza, 2008), as well as the literature review (Buteau et al., 2010), identified many issues relating to the integration of CAS into the post-secondary mathematics classroom. In what follows, we contrast some of these results.

The literature review identified 16 separate issues relating to CAS integration at the post-secondary level, organized into three categories of technical, financial and pedagogical types of issues.

The issue of *assessment* was examined in detail by both studies. Out of the 1103 practitioners who responded, the international survey (Lavicza, 2008) reported that "there were relatively few written responses (10) that referred directly to assessment" (p. 176). In contrast, the literature review reported that assessment was the most widely stated issue with 12 out of 204 papers discussing it (see Fig. 1). Some of the written comments from the international survey (Lavicza, 2008) did demonstrate that instructors were concerned about this matter. One practitioner argued that,

> The two major issues have been assessment, which must of course reflect the way that the students have been learning, and must incorporate technology in some ways, and curriculum content, which has been the subject of considerable review. The assessment has been the most controversial. (pp. 176–177).

Lavicza (2008) noted in the international survey that "assessment is a critical and polarizing issue in CAS-assisted teaching and learning" (p. 176). This was echoed in the literature review. In response to Wilson and Naiman's (2004) study on post-secondary student calculator use, Tunis (2004) wrote a letter to the editor asking, "How can my examinations build upon my students' ability to select the appropriate technologies (during the exam) while still getting at their understanding of mathematics?" (p. 1). He concluded that he was "not sure we can give the same kinds of exams that we have traditionally given" (p. 1).Fig. 2 However, replying to Tunis' letter, Wilson (2005) argued,

> Actually, the traditional exams are designed to test the students' understanding and problem solving ability with the new material. The true test of whether technology in the classroom has succeeded is whether it improves the students' performance on traditional exams. YOU CANNOT REDEFINE MATHEMATICS! (p. 2)

In contrast to Wilson, the international survey (Lavicza, 2008) stressed that examinations must be adapted to the technology. "CAS use without assessment practices was not taken seriously by students. As Dr. Tall in Phase-I put it: 'If it is not in the exam it is not on the course'" (p. 176). While the literature review lacked an explicit endorsement of mandatory assessment with CAS integration, the corpus complemented this argument with instances of assessment of students' CAS skills. Keating (2004), for example, reported that at Eastern Connecticut State University, "as a means to assess student competency, a *Maple* 9.5 Mastery Test was administered and graded" (p. 139).

The international survey identified several difficulties with CAS relating to *students failing to achieve learning objectives*. Lavicza (2008) explained that, "Some mathematicians believed that learning with computers discourages students from understanding mathematical concepts and that they will only seek results from machines without understanding their meanings" (p. 218). Several papers in the literature review expressed similar concerns. Dana-Picard and Steiner (2003), for example, agreed with the international survey, noting that if a

> student obtains an answer from a CAS and uses it as it is, without gaining any insight into the mathematical processes involved and probably not realizing the existence of other (superior) methods or algorithms different from the standard ones, he/she cannot check the correctness of the solution and certainly has absolutely no chance to invent his/her own methods. (p. 1)

[5] Using Sections,9.4, 9.6, 10.6 and 12.7 of Lavicza's study.

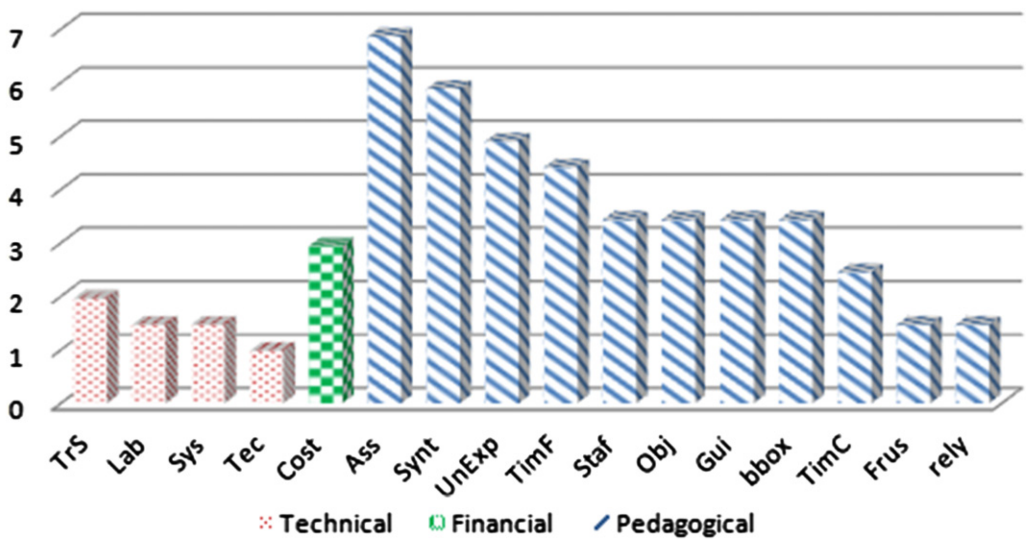

Fig. 1. Issues in CAS integration in the literature review. (Buteau et al., 2010) Trouble Shooting (TrS), Lab availablility (Lab), System Requirements (Sys) Need for Technical Support (Tec), Cost of CAS (Cost), Assessment (Ass), Syntax (Synt) Unexpected Behaviour of CAS (UnExp), Faculty Time demands (TimF), Limitations of Trained Staff (Staff), Failure of Students to Achieve Learning Objectives (Obj), How Much Guidance (Gui), Black Box nature of CAS (bbox), Time demands in Course (TimC), Student Frustration (Frus), Student Overreliance on CAS (rely).

In both studies, practitioners emphasized the need for students to *understand basic mathematical concepts before using CAS*. (e.g., Lavicza, 2008, p. 219). Herwaarden and Gielen (2001) endorsed a process of redoing assignments first attempted by hand. They argued that “often students show a lack of conceptual insight while using the computer algebra system, because, as it seems, they don’t incorporate in a right way the computer techniques into their mathematical way of thinking” (p. 1).

Overreliance on CAS was a concern in the international survey and was echoed in the literature review. it was noted in Lavicza’s international survey that hand calculations are particularly valued by practitioners.

> Students should not learn to rely on CAS for solving problems. Working through a solution by hand forces them to think about the underlying physical meaning of what they are doing, making sure units work out, etc. This is often lost if they are permitted to use CAS. (2008, p. 219)

The literature review identified student overreliance on technology as an issue (Buteau et al., 2010, p. 61). It did however find a point of contrast. Berry (1999) wonders “how much of these activities need to have been carried out by hand before we use the technology? This question is a constant dilemma to me" (p. 76). Berry (1999) further writes that he “often feel[s] pulled towards the view that ‘students should be able to do this or that using pen and paper techniques’. Inevitably there is the feeling that if I can do it ‘by hand’ then the students should also!” (p. 74).

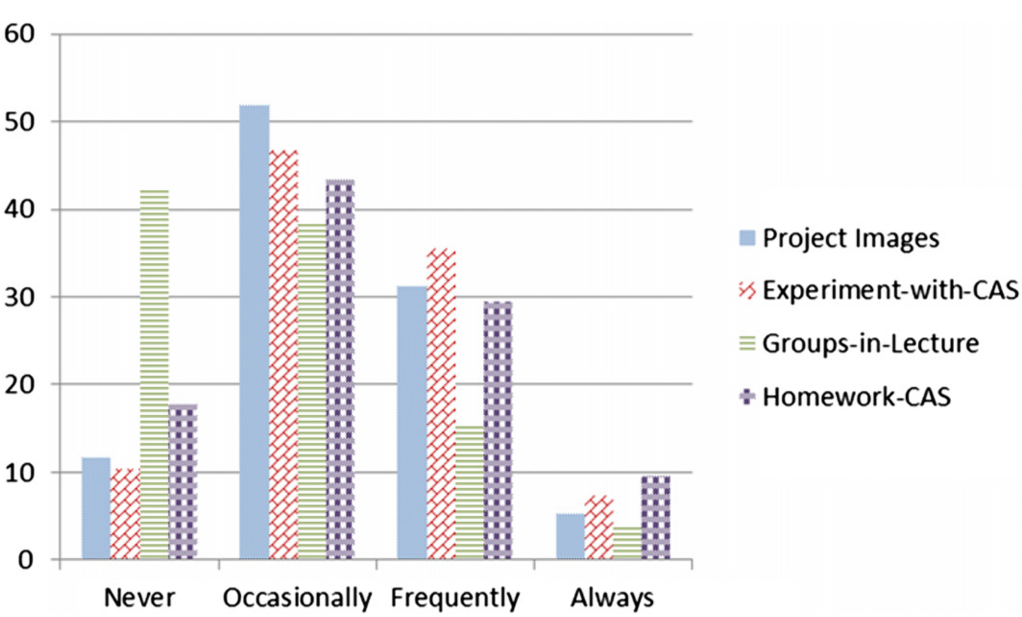

Fig. 2. Uses of CAS-based technology in the international survey, $n = 491$ (Lavicza, 2008).

Instructors also faced obstacles in CAS integration in their teaching. The international survey (Lavicza, 2008) found disagreement amongst practitioners regarding whether or not to *dedicate class time to CAS instruction*: “Some argued that CAS programming should be learnt by students independently and out-of-class”. Other practitioners “argued that class time should be dedicated to teaching programming” (p. 221). The literature review reaffirmed this conflict. Little (1994) wrote that “class time should still be devoted to mathematics and the use of the calculator can stay on the general level with specifics left to handouts for the students to work through on their own time (often in advance)” (p. 5). Barton (1995) reported that a large factor in the success of a CAS integration was a "willingness on the part of the teacher to risk the limited class time in demonstrating concepts with the calculator" (p. 5).

Time-related pressures were also recognized as problematic outside the classroom. The international survey (Lavicza, 2008) noted that “the time required for preparing CAS-enhanced classes was overwhelming” (p. 225). The literature review revealed similar sentiments. Wrangler (1995), for example, argued that near constant improvement is needed in lab experiments, and stressed that for faculty there is “no resting on laurels” (p. 8).

Another point that the international survey emphasized is the notion that *CAS has to be used appropriately*. Lavicza (2008) wrote that “CAS should not be integrated into the curriculum without clear objectives and careful thinking” (p. 222). As a related example, he quoted a practitioner who warned,

> I believe that there is far too much effort being put into using CAS wherever possible in curricula. Instructors should know what material they want their students to understand, and then think of ways that CAS might help. Instead, many instructors decide that they want to use CAS in certain ways, and then meld the other material around that. CAS is very flashy and entertaining, but that doesn’t always translate into being educational. (p. 222)

Though not directly addressed in the literature review, the risk that CAS integration, if poorly implemented, could contribute little or nothing to student intellectual growth certainly merits contemplation. While both studies, as will be addressed in more detail below, found that mathematicians struggled with poor student attitudes, neither student nor instructor gain any tangible advantage through mere amusement, if that is all that the planned CAS use provides. A successful integration thus demands prudence and reflection on suitable uses of CAS to avoid such pitfalls.

When compared to the international survey, the literature review only narrowly identified the issues such as *limitations of trained staff* and the *need for practitioners to troubleshoot issues* (Buteau et al., 2010, pp. 58–59). The international survey argued more broadly for the need of a larger departmental role. Lavicza wrote that “departments could play an important role on mathematicians’ decisions on technology integration. . . Furthermore, departmental decisions and support seemingly have some effect on the use of CAS in teaching” (2008, p. 228).

Access to technology presented additional problems. Both the international survey and the literature review identified lab availability as a barrier to CAS integration. In the international survey, Lavicza (2008) proposed that “this might be a result of lack of funding, but it is possible that the administration does not pay attention to these needs” (p. 233). The international survey explored this issue further, with Lavicza noting that “even if the classroom infrastructure was available, its design may not be appropriate for offering CAS-assisted classes” (2008, p. 233). Lavicza (2008) did comment that lab availability may not be a significant issue, conjecturing “that universities are reasonably well equipped and well supported by technology resources” (p. 284). He further argued that “it is likely that in a few years students will own a mobile computer with CAS applications” (p. 280) and that such availability may further mitigate problems involving CAS use in computer labs and lecture halls (p. 284). While the literature review did not have practitioners commenting on the potential of hand-held computing and CAS, it did find that lab availability was a decidedly minor issue (see Fig. 1) encountered in less than 2% of the paper corpus, supporting Lavicza’s contention.

The *cost of commercial CAS software* was identified both by the international survey and the literature review as another prohibiting factor in CAS usage. The international survey noted that some practitioners attempted to integrate open source technologies as a means of controlling these costs: “Exorbitant cost (*Mathematica*) and bothersome licence issues (*Mathematica*, *MatLAB*) have kept me away from the commercial ones. I have been trying to work with systems available freely in the academic environment such as the *C* interpreter from Soft Integration or *GAP*, etc” (Lavicza, 2008, p. 233). The literature review also identified practitioners who addressed the cost issue by opting to use free software. Hohenwarter, Preiner, and Yi (2007) quote a practitioner who endorsed *GeoGebra* for these reasons: “One of the big factors was that it’s free, so there’s nothing stopping students from putting on their own machines and for our IT people from installing it on any campus machine I want” (p. 89). This link between cost and access is emphasized in the international survey, with Lavicza (2008) remarking that “students often can’t access CAS at home or afford buying student licenses” (p. 234).

CAS itself may pose an obstacle to integration. Lavicza in the international survey argued that “the major hindrance to integration into the classroom is the complexity of *CAS syntax*” [emphasis added] (2008, p. 229). The literature review supplements this statement by identifying syntax as the second most widely reported issue (Buteau et al., 2010, p. 60). A typical example was Cherkas (2003). He provided commentary by one of his students who, frustrated by the inclusion of CAS in the course, complained “*Mathematica* would cause a lot of problems. If I make a mistake in the syntax, I couldn’t do my work” (p. 231).

The international survey further identified *problems with CAS software*. It reported practitioner dissatisfaction with the “constant upgrades of commercial software packages that are often not compatible with previous versions” (Lavicza, 2008, p. 229). Interface too was a concern. “Providers should continue to find more user-friendly CAS interfaces for beginning students” (Lavicza, 2008, p. 230). The literature review did not find practitioners who discussed problems with software upgrades, though it identified a related concern with the need of practitioners to perform general troubleshooting of CAS code. While the survey did not address mathematicians’ adaptations to the interface issue, practitioners in the literature review adopted various methods to adapt to this problem. For example, Schlatter (1999) designed GUIs for *MATLAB* so that the students would have a small "learning curve," and similarly May (1999) urged that worksheets created for student use should be "user-friendly" requiring minimal knowledge of *Maple* syntax for use.

According to the international survey, *student preparation and attitudes* were of some concern to practitioners. Lavicza (2008) reported that “the decline of students’ mathematical knowledge made some mathematicians believe that instead of incorporating CAS and teaching advanced topics, they have to teach basic mathematical concepts” (p. 232). In contrast, the literature review found some mathematicians who saw poor student performance as a catalyst for integration. Yates (1994), for example, noted:

> Watching 50–75% of my students get a D, F, or W convinced me that the old way does not work. . . . I had read for some time about the paradigm of a lean and lively calculus: more technology, less lecture, fewer but richer topics, and cooperative learning. As our department had just mandated graphing calculators for all pre-calculus and calculus courses . . . I resolved to try something. (p. 1)

While both studies identified barriers to integration, a majority of papers in the literature review (63%) did not report on issues relating to the use of CAS in teaching. These findings are in parallel with the results from the international survey. In discussing these barriers, Lavicza concluded that "mathematicians were not excessively concerned about difficulties [that] arose in relation to the integration of CAS into teaching" (2008, p. 234).

3.1.2. Mathematical literacy and CAS

The international survey examined the beliefs and conceptions of mathematicians and the notion of mathematical literacy with respect to basic academic mathematical knowledge. CAS is viewed by some mathematicians in the international survey as "*an important tool*" *for performing mathematics* [emphasis added] (Lavicza, 2008, p. 212). Another practitioner from the international survey is quoted as saying "I use it in research, shouldn't the students?" (p. 213). In the literature review, we found that some practitioners concurred with this idea. Savari (2005) reported that "from these examples we can conclude that computer algebra systems are often suitable for demonstrating, realizing, and initiating discoveries not only for researchers, but also for the students" (p. 10). Avoili (1994) explained that among other purposes that "*Maple* is used . . . to illustrate to the student that computer software is a valuable tool in a mathematician's tool box of methods to solve problems" (p. 1).

The changes to the world that have been brought about by the technological revolution have not left the post-secondary classroom untouched. The international survey (Lavicza, 2008) noted that "some mathematicians stated that *it is inconceivable to teach without technology at our time*" [emphasis added] (p. 213). This was supported by the literature review: Putz (1995) after integrating *Maple* in his multivariate calculus class, wrote "I'm convinced that *this* is the way to teach the course. I can't imagine teaching it now without a computer algebra system" (p. 4).

As CAS-technology matures, it may be that *it will undergo phases of acceptance* similar to calculators. In the international survey, Lavicza argued that "computers are rather new elements of mathematics teaching and learning and they are slowly becoming accepted in teaching mathematics. Calculators produced similar controversies when they were introduced into teaching mathematics" (2008, p. 213). This notion was also echoed by Barton (1995) in the literature review:

> It is during the third or final phase of "acceptance" [with CAS integration], that the new methods of teaching through integration of the technology will become the preferred way of teaching by the mathematics community as a group. . . . However, there is still much that must be done before the third phase can become a reality. (p. 1)

The literature review had additional complementary results. Turegun (2006) expanded on the acceptance theme, arguing that one can substitute CAS calculators for statistical distribution tables:

> In the past every published trigonometry textbook contained some tables listing the function values of sine and cosine functions for various increments of the argument. With the wide spread usage of scientific calculators, however, these tables gradually have disappeared from the trigonometry textbooks. If one does not see trigonometry textbooks published with the function value tables anymore, why should one still see introductory statistics textbooks with various critical value distribution tables. (p. 1)

A mathematician in the international survey (Lavicza, 2008) generalized Turegun's point, writing "It's the future of computation. The use of calculators isn't questioned anymore; computers will be part of teaching" (p. 213). The literature review did not reveal a statement by a practitioner that was as explicit. However, the possibility that it could be the future of mathematics was explored: "A major topic of interest concerning the future of mathematics education in recent years has been the use of calculators and computers as tools in the teaching and learning process" (Barton, 1995, p. 1).

CAS was viewed in both the international survey and the literature review as *potentially providing a need/benefit in the 21st century*. A mathematician from the international survey (Lavicza, 2008) reported that,

> Technology is part of life in this century, and we do our students a disservice if we do not encourage them to explore its impact on mathematics - which is undoubted. If it has changed how I do mathematics, why should not my students experience that? (p. 214)

The literature review supported this assessment. It also found that practitioners were contending that technology was a necessity in this new century. Wu (1995) arguing for CAS integration in the classroom concluded that

> With added pressure from the rapid development of information super highway, it is even more demanding to train our students to think clearly, critically, constructively, and creatively about the problems they might encounter in the real world. It is our job to help students to gain the ability that will enable them to use mathematical methods and tools whenever they seem appropriate and helpful. To this end, computer-oriented mathematics courses, focusing on [cooperative] learning, problems solving, and investigative learning and writing are an important part of the education for our students. (p. 4)

Lavicza's international survey (2008) examined the relationship between CAS in service courses and CAS in courses for mathematics majors. He wrote that

> mathematicians emphasized the importance of CAS knowledge for students studying science and engineering courses while they were less convinced by its value for students majoring in mathematics (p. 290).

This result was not corroborated by the literature review which found that integration "occurs most frequently in courses for mathematics majors, as opposed to service courses designed for non-math majors" (Buteau et al., 2010, p. 62). However, Lavicza (2008) also reported that 94% of mathematicians who taught mathematics majors also employed CAS in teaching students majoring in mathematics, the largest proportion (p. 182). It can be conjectured that while mathematicians may have reservations about the value of CAS for mathematics majors, both studies suggest that such reservations have not hindered integration for mathematics majors.

The international survey reported that 90% of CAS users employed computer-based CAS in their teaching, 29% utilizing hand-held technologies with 28% of mathematicians using both platforms. (Lavicza, 2008, p. 162). The literature review exhibited similar results with 59% majority of papers detailing only computer-based CAS use, 30% using hand-held technologies and 10% of papers featuring both technologies (Buteau et al., 2010 p. 58). While there is a discrepancy on sole use of hand-held CAS in the classroom, this may be a result of bias contributed by the 46% of papers were only presentations of examples[6] and thus not detailing all technology used. However, one may still conclude that both studies suggest that a large majority of CAS is computer-based in post-secondary education.

In both studies, practitioners have discussed *how CAS has influenced their practice and teaching of mathematics*. While the use of CAS in the post-secondary classroom remains very controversial, the survey suggests, and the literature review supports the notion that there is a healthy and ongoing debate over what roles such technology should play in this environment.

3.1.3. Diverse use of technology by practitioners

In the international survey, Lavicza (2008) highlighted the fact that “mathematicians integrated [CAS-based] technology into their teaching for different purposes” (p. 165). The international survey identified three primary uses of CAS by practitioners. The use of CAS to project images and provide graphical interpretation of mathematical concepts was used by nearly 90% of mathematicians surveyed (Lavicza, 2008, p. 166). The literature review supported this result since it also had *visualization* as the second largest purpose with 59% of papers discussing it in some fashion (see Table 1). In the international survey, a number of mathematicians “mentioned that when drawing on the board becomes complicated and time-consuming CAS can be particularly useful” (p. 167).

The literature review found complementary results. Dana-Picard and Steiner (2003) reported that teaching with technology “offers an opportunity to develop more complicated examples than in the blackboard and chalk teaching, examples which would be impossible to solve without software” (p. 2). The idea that students should actively use CAS visualization capabilities rather than passively watching a CAS-based lecture was also expressed by practitioners in the literature review. Spitznagel (1994) cautions that,

> if you have a TI ViewScreen, resist the temptation to turn to it too quickly, as this will discourage students from using their own calculators in class. Instead, ask the students to use their calculators to do something—and then, after 15 s or so, follow up with the same thing on the ViewScreen, so they can check their work for correctness. (p. 1)

Based on the international survey, Lavicza (2008) noted that students could sometimes draw improper conclusions from CAS-based visual instruction. He wrote that “images presented in these lectures were viewed favourably by both teachers and students, but careful considerations should be made for their use to be effective in teaching” (p. 168). While the literature review did not find specific agreement with this, it did identify the issue that students will sometimes fail to achieve pedagogical objectives (Buteau et al., 2010, p. 59), which corresponds to Lavicza’s assertion.

Experimentation and discovery activities were found in the international survey to be the most popular use of CAS by practitioners, with 60 instances of written comments (2008, p. 168) and 90% of mathematicians reporting use.

The literature review agreed with the survey, finding experimentation to be a significant purpose as well, with 63% of papers mentioning it. Lavicza (2008) noted that “several mathematicians explained that they could also incorporate science-like laboratory components into classes in which students can experiment with concepts and then develop rigorous reasoning of these concepts” (p. 168). This approach was commonly reported in the literature review. For example, in Kramer’s number theory course at Queens’ College, “*Mathematica* was introduced . . . during the Spring 1997 semester, to encourage the discovery of mathematical ideas through guided experiments” (1997, p. 1). After instructing students the basics of *Mathematica* programming and providing initial modules of code, Kramer explained that “by modifying these simple modules, students were able to conduct experiments and test conjectures” (p. 1).

The literature review concurred, with Xie (1994), arguing that,

> The rapid development of computer technology has made it possible to teach mathematics by developing interactive mathematics laboratory. A mathematics laboratory is equipped with new generation of computers and other accessories. The "experiments" in the laboratory is to explore interactive electronic texts, that can be described as computer documents from which symbolic, numeric and graphical tools can be invoked together with features that support experimenting with as many examples as one wants to understand and explore ideas. (p. 1)

Lavicza noted another distinction, which was absent from the literature review:

> Most importantly from mathematicians’ point of view, these kinds of [experimental] activities can be similar to mathematical research and offer a valuable approach to teach mathematics at universities. The emergence of computers and mathematical software further enabled mathematicians to engage students in research-like activities. (2008, p. 169)

More than 80% of respondents from Lavicza’s international survey encouraged or required the use of CAS in homework and assignment problems. Corresponding results from the literature review were much smaller, with only 16% of papers mentioning it (see Table 1). Such discrepancies are not surprising given that the international survey directly asked practitioners how they used CAS and in what ways, while the emphasis of the papers analyzed in the literature review corpus is not necessarily on rigorously documenting actual classroom practices. It is rather focused on reporting examples of how CAS is used to solve specific mathematical problems. While the differing data sources do give, in this particular case, conflicting results, one may infer that this apparent conflict stems from the different methodologies used in the research and that a more complete picture can emerge from considering this context.

In the international survey, Lavicza found that mathematicians had their students use CAS for *homework* in different ways. In a written comment, an instructor explained that “when teaching numerical methods in calculus and differential equations, I ask my students to do certain homework problems by hand and others with a CAS” (Lavicza, 2008, p. 172). The literature review’s findings support this result with some practitioners adopting a mixed approach with CAS. In her first-year calculus class, Power (2000) instructed her students to use both

[6] See Table 1, Buteau et al., 2010, p. 57.

Table 1
Uses of CAS-based technology in the literature review (Buteau et al., 2010).

Uses of CAS-based technology	
Experimentation and exploration	63%
Visualization	59%
Real and complex problems	50%
Homework and assignment preparation	16%
Group work	9%
Conceptual discussions	8%
Motivation	8%
Check solutions and problems	3%

a graphical calculator and hand-written calculations to investigate curves. She noted, “First you will graphically investigate absolute extrema of that function on the closed interval domain. Then analytically you will solve for the absolute extrema of the function” (p. 1). Other practitioners in the literature review had students redo assignment questions with CAS. Herwaarden and Gielen’s study (2001) instructed their students to "solve this system [of linear equations] with paper-and-pencil. Subsequently we let them solve the system with computer algebra. . . and compare the answer with the result they had obtained by hand" (p. 2). It was the goal of their study to have students establish similar links with CAS that a student has with hand-written calculations:

> After some similar exercises to obtain more practice and to master the process, more difficult assignments have to be made by the students using computer algebra. These may be extensions of former assignments that are too elaborate or demand too much technical skill to treat without computer algebra. (p. 2)

The use of electronic assignments was reported in both studies. The international survey noted that “out-of-class exercises that encourage or require the use of CAS are increasingly utilized in various mathematics courses” (2008, p. 172). This idea was also echoed in the literature review. To facilitate online labs for students, Dogan-Dunlap (2003) chose *WebCT* along with *WebMathematica* as his technological infrastructure. As well, the use of electronic documents for student laboratory work and learning was discussed in the literature review. For example, Alexander (1998) used *Studywork* files to present lecture topics, and May (1999) created structured *Maple* worksheets to explore topics in computer laboratories and student homework.

The literature review found that the use of CAS to solve *realistic or complex problems* was reported in 50% of papers in the corpus. The international survey, while it did not find that this was a major purpose, argued that “CAS could supply the computational power to carry out large-scale calculations and in this way ‘real-world’ problems are not only, as often criticised, simple problems with some ‘made-up’ commentary” (Lavicza, 2008, p. 169). The literature review also identified practitioners that saw this potential. For his course in partial differential equations, Xie (1994) used

> realistic applications . . . from a variety of areas to introduce and motivate mathematical concepts. For example, the animation of the vibration of a violin string (one dimension) or a stretched membrane, such as a drumhead (two dimensions) will be presented to derive the wave equations (p. 2).

Xie also agreed with the international survey result that CAS may assist in calculations, noting that “in short, the computer removes computational burdens, allowing the student to work on more essential contents” (p. 7).

Both the literature review and the international survey had practitioners commenting on how some mathematics they teach is only feasible with CAS. In the international survey, Lavicza noted that “problems can be formulated that would not be possible without CAS” (2008, p. 170). Practitioners in the literature review agreed. When discussing a complicated optimization problem involving a system of equations and finding the limit of a multivariate function Kreczner (1995) wrote that,

> this problem [is not] as simple as it might seem at the first sight, to carry out this computation, even for simple equations $F(x, y) = 0$, is very laborious and tedious, or even impossible to do especially for most students. There are three main reasons for these, the system of equations… is to [too] difficult or impossible to solve, the expression… is lengthy, and thus the computation of limit… is not clear. In contrast, with the [sic] *Mathematica* all these difficulties might be avoided, especially if the equation $F(x,y) = 0$ is relatively simple. (p. 3)

The international survey (Lavicza, 2008) also reported that CAS use can decrease the time teachers and students spend on *lengthy computations* and can thereby increase the amount of class time dedicated to discussing more complex theories and *focussing on important ideas* (p. 170). Similarly, the literature review found many practitioners who agreed with this. For example, Avioli (1994) reported that in his class “*Maple* is used to enhance the teaching of the calculus . . . to solve more ’real’ problems which involve hard, long, tedious calculations [and] to permit more time spent on concepts rather than on calculations” (p. 1). According to the international survey results, Lavicza revealed a complementary result for mathematicians’ students: Integration not only enables class time to be focused on theory but it also permitted undergraduates “to work on problems and develop conjectures [while] preparing for lessons” (Lavicza, 2008, p. 170).

CAS can allow mathematicians the opportunity to have their *students work in teams instead of just as individuals*. The international survey (Lavicza, 2008) found that practitioners could have their students work together in lecture or merely in out-of-class settings (p. 166). Results from the literature review show a different classroom practice, yet arguably involving the same idea. The corpus contained more than a few examples of practitioners moving from away from traditional instruction methods into a computer lab where "students work in small groups of three with the instructor acting as a facilitator" (Alexander, 1998, p. 2). Further, Lavicza (2008) stressed that “group/team work is becoming an important element of mathematics teaching at universities” (p. 167). Several practitioners in the literature review reported the use of collaborative learning. For example, Yates (1994), attempting course redesign with technology, noted that “several colleagues were engaged in group work and student projects in their courses” (p. 1).

Lavicza’s international survey reported that CAS is used indirectly by some practitioners to *develop problems and examples* (2008, p. 172). Unsurprisingly, due to the fact that nearly all of the papers did not discuss the day-to-day activities of practitioners, this was not mentioned very often in the literature review. Xu (1995) mentioned using CAS to help determine assignment problems for his class that were difficult to solve via a graphing calculator, but relatively easy to solve with hand-written calculations. The international survey provided additional data on practitioner-centred uses, reporting that some mathematicians used “the graphical and computational feature of CAS. . . to check solutions [of students]” (Lavicza, 2008, p. 172).

The use of CAS to *motivate students* was a minor purpose in both the international survey (2008, p. 170) and the literature review (see Table 1). Notwithstanding, some mathematicians did argue in the international survey that CAS may help make classes more fun or more interesting (Lavicza, 2008, p. 171). The literature review found there were mathematicians who agreed with this sentiment, as expressed, for example, by White (1997) who noted:

> Traditional methods of teaching do not relate mathematical problems to the real world, help students think about realistic situations, or help students to generate and pose their own solutions. As a result, students may become unmotivated and unconnected, developing an overall negative attitude towards mathematics and technology. (p. 3)

In this section, we have examined the results of both studies through Lavicza’s eight purposes of practitioner use of CAS. The international survey’s eight purposes are largely supported by the complementary results from the literature review.

3.1.4. The potential benefits of CAS integration

Lavicza’s international survey reported that practitioners promoted the potential positive effects of technology integration for their students in their courses. Such sentiments were, as well, present in the literature review paper corpus. These include: gaining additional skills needed in the 21st-century workplace, aiding student comprehension of mathematical concepts, reaching out to weaker students, motivating students to explore more mathematics courses, exploring more difficult mathematics earlier in the curriculum, and, introducing a student to a potential career in research.

The notion that *CAS can possibly aid a student at the post-graduation stage* is discussed in both studies. The international survey explored how this may provide momentum for integration: “As students need CAS for their career, mathematicians and the department are likely to integrate more CAS into teaching” (Lavicza, 2008, p. 214). Practitioners identified by the literature review also recognized the potential of CAS regarding employability. Challis (2001), for example, explaining the reasons of a complete integration at Sheffield Hallam University (p. 1) wrote that "employability of our graduates is high on our list of aims, and so key or transferable skills are important." These skills were also recognized as important by Pollack-Johnson and Borchardt (1995), who noted that

> because students are working on projects about which they care deeply, they really learn the process and concepts. This makes it possible to later transfer the knowledge and skills to business and career problems about which they will have a similar passion, since their salaries and careers are likely to be at stake. (p. 4)

Thus, this conjecture by Lavicza that CAS integration may be aided by students’ future employment needs is reinforced by the literature review’s findings on CAS-related transferable skills.

Some mathematicians believed that *CAS can also aid in their students’ comprehension of mathematical concepts*. The international survey (Lavicza, 2008) results included a quotation by a practitioner who emphasized that “the use of CAS must lead to student understanding of concepts that might be difficult to learn without CAS; CAS can be used to study problems or concepts that would be difficult without using it” (p. 219).

The visualization capabilities of CAS and their ability to easily link these graphics to other mathematical representations have also been discussed by mathematicians in the survey: “[I use CAS because of] the ability to connect visualization and symbolization,” writes an international survey participant (Lavicza, 2008, p. 167). Practitioners in the literature review argue that when students carry out experimentations through CAS, this can aide in their mathematical learning. Challis (2001) noted that "CAS can make a contribution here by expressing a mathematical idea as an object which is then open to discussion and interpretation - a sort of conversation piece" (p. 6). Thus the literature review coincides with Lavicza’s findings that practitioners may be able to use these visualization capabilities to aid student’s understanding of traditional, lecture-based course material.

Lavicza noted in the international survey that “mathematicians greatly emphasized the importance and promotion of students’ conceptual understanding through CAS-assisted activities” (2008 p. 220). While the review contained many positive reports on student achievement as a result of CAS, Emese’s study (1994) found that there was no statistical difference in conceptual understanding between a group of students using graphing calculators with the discovery method, a group of students who used graphing calculators without the discovery method, and control group with no hand-held CAS permitted. Other practitioners disagreed with Emese’s findings. White (1997), for example, reported that “Ganguili concluded that the significant difference in the final examination indicated that students had acquired and retained conceptualizations of algebra better in the treatment group than in the control group” (p. 13). It should be noted that Emese study does not necessarily contradict Lavicza’s findings, as there were practitioners that participated in the international survey who were sceptical of the educational value of CAS as well.

In the international survey, Lavicza (2008) emphasized how CAS may *motivate students* and thus be beneficial in teaching (p. 222). He pointed out that it may help make classes more interesting, engage students in the classroom, enthuse students to learn mathematics, and enable them to become more active learners. He further reported that “some mathematicians indicated that CAS could offer opportunities to engage students with weak mathematical abilities and this could widen access to mathematics” (p. 219). The literature review found practitioners who concurred with these ideas. White (1997) noted that "creative applications of technology can restore much of the thrill of exploration by giving even our less skillful students tools to take them where they could not have easily gone before" (p. 9). The review also identified how a change in pedagogy could be seen as providing opportunities for students. After shifting the majority of instruction time to interactive labs for his partial differential equations course, Xie (1994) argued that his “students are invited to become active learners, rather than passive recipients of lectures. Students, not professors, are more actively involved in seeking solutions to the questions and problems. As a result, they are more motivated and enthusiastic in participating in their learning process" (p. 7). In the literature review, motivation of students is also considered beyond the individual course. When Georgia State University integrated CAS into their college algebra course, Alexander (1996) reported that the goal was to encourage students to take further math courses. She explained that her departmental philosophy is that "college algebra should be a pump not a filter" (p. 2). Clearly this proposition that CAS can help inspire students to pursue further math courses is an extension of Lavicza's findings.

There is also a perception by some practitioners that CAS can permit their students to work with *more complex mathematics earlier in their mathematical career*. The literature review reported several examples of this particular claim. While exploring parametric curves in Maple with his multivariable calculus class, Putz (1995) commented that "an advantage of using the CAS is that students can start analyzing some interesting curves right away—much earlier than we would have expected them to do just plotting by hand" (p. 2). This supports the findings of the international survey which reported on mathematicians who shared this belief. For example, one mathematician wrote that "CAS-related teaching should be used to encourage students to explore mathematics. More sophisticated ("real life") math becomes accessible via CAS: e.g., linear programming can be taught in an introductory linear algebra class" (2008, p. 169). The international survey further expanded on how CAS may open doors to higher learning for students. For example, a practitioner noted that "richer content areas can be explored that are too daunting with paper and pencil" (p. 168).

One potential benefit of CAS integration solely reported by the international survey was how *experimental activities may help prepare students for a career in research*. Lavicza (2008) wrote that "the emergence of computers and mathematical software further enabled mathematicians to engage students in research-like activities" (p. 169).

Lavicza (2008) also reported that *using CAS may benefit practitioners themselves*: "Some felt that CAS enhanced their teaching practices and in this way not only contributed to students' learning but also to their own satisfaction in teaching" (p. 221). The literature review results bore similar messages. Yates (1994), commenting on his students' success after completing a CAS-based project reported that he "was pleased and impressed with the students' work on this project" (p. 3).

3.2. Comparing conclusions of both studies

Lavicza's international survey presented many quantitative results regarding mathematicians' practices, beliefs, and concerns regarding CAS use in post-secondary mathematics education. However, the international survey goes much beyond this. In comparing individual integration in Hungary, the UK, and the United States, Lavicza writes that his analysis "failed to detect substantial differences among mathematicians' conceptions in different countries and on the influence of teaching traditions on CAS integration" (2008, p.307). He concludes that mathematicians' views on integration and their attempts to integrate technology in their classrooms may have an international character. Also the international survey involved mathematicians who used CAS and those who did not, allowing a comparison between the viewpoints of both groups. Lavicza found that "non-users of CAS were more concerned about the problems hindering CAS integration, while users of CAS had more elaborated conjectures related to teaching and learning issues" (2008, p. 241). Furthermore, in the international survey, factors that influenced CAS integration by individual practitioners were investigated. Lavicza concluded that "according to the SEM model […] participants' use of CAS in teaching was most highly influenced by their use of CAS in their own research. In addition, […] more than 80% of mathematicians who used CAS in their research also utilized it for teaching." (2008, p. 288).

As part of his conclusions, Lavicza proposed five direct implications of his comprehensive research (Lavicza, 2008, pp. 307–310). The first implication stresses the need to holistically examine technology integration in post-secondary mathematics departments. Indeed, Lavicza notes that "due to the complexity of this issue and the quantitative nature of this study, further in-depth investigations should be carried out to complement findings of this study" (2008, p. 308). The literature review itself as a thorough study contributes to this first implication. In what follows, we briefly examine complementary results stemming from the literature review in light of the other four implications.

Two of the emerging themes from the literature review were the documentation of innovative and unique examples of CAS tasks, and the documentation of those tasks that were most often repeated throughout the corpus. The literature review stresses that "in many instances within the reviewed corpus of papers, instructors would refer to common uses of CAS described in their work as being *new*." This fact may underscore a possible deficiency in rigour by journal editorial staff and/or reviewers in accepting papers that make these types of novelty claims. More importantly, however, it also emphasizes the need for better communication and shared materials/resources between mathematicians" (Buteau et al., 2010, p. 64). These outcomes directly support Lavicza's second implication of his study, where he emphasizes the need to "review, organize and collect existing resources and document exemplary practices" (p. 308) regarding CAS integration. Thus the literature review also aids in this particular goal. In his third implication, Lavicza proposes that there is a need to study potential means of providing CAS-related support for mathematicians (p. 309). He argues that "encouraging the development of supportive and collaborative working environments could be a valuable contribution to the integration of technology into mathematics teaching" (p. 309). The identification of repetitive examples in the literature review reinforces Lavicza's conclusion that there is a need for a more collaborative working environment.

The literature review also examined the various degrees of integration at post-secondary institutions. Buteau et al. (2010) noted that

> 67% of the reported CAS-based technology was liked to use in a single course, or in other words, by a dingle practitioner…. In contrast, very little indication of program-wide, or systemic, CAS use was reported in the papers examined. Such a systemic integration in the curriculum would require, among other facts, an initial consensus among colleagues in a mathematics department – a major step representing a significant challenge in of itself. (p. 64)

Consensus surrounding the importance of technology use in teaching among the faculty in a mathematics department could clearly be aided by a supportive and collaborative working environment. In addition to supporting Lavicza's third implication, departmental integration of technology use could also be aided through the nurturing of "collaboration between mathematicians and mathematics educators" (p. 309), his fourth implication resulting from the international survey. This implication may also be partially addressed through analysis of the papers within the literature review corpus that have been written by educational researchers (Jarvis, Lavizca, Buteau, & Marshall, in press). Finally, note that Lavicza's fifth and final implication, the need to tackle problems in regards to students' transition from school level to post-secondary level mathematics, lies beyond the scope of our literature review.

Several findings by Lavicza's international survey support the outcomes of the literature review. Although 88% of the corpus was comprised of American papers, Lavicza's conclusions regarding the international nature of post-secondary mathematics education (2008, p.

307) include his argument that this would not likely contribute any significant bias affecting the outcomes of the literature review. The similarity in results between the literature review and the international survey suggests that other sources of bias may be of a minimal nature as well. Although we were purposefully limited to only two journals and to two sets of conference proceedings in our study, and although mathematicians may have chosen to report only things that were of personal significance to them in their papers, the agreement of the literature review findings with those of Lavicza may indicate that these particular factors did not largely influence the final results. Furthermore, although the literature review study involved contributions from mathematicians over a much larger timeframe (1990–2008) than the “snapshot” international survey of 2006, the complementary and similar results from both studies may suggest that this time difference was also not a large factor.

One advantage that the international survey had over the literature review was that the views of non-CAS users could be sampled and compared with those who did use CAS in their teaching. In reviewing various obstacles to integration, Lavicza conjectured “that CAS users have more elaborate conceptions about teaching and learning issues and turn their attention toward pedagogical questions whereas non-users are more concerned with the difficulties that could prevent CAS integration into teaching.” (2008, pp. 240–241). Given that the literature review, with only one exception, was comprised of papers written by CAS users, the identification of assessment as the primary issue of concern by CAS mathematicians may perhaps be supported by this fact.

4. Final remarks

Our comparative study suggests that the results of our literature review strongly support those of Lavicza’s international survey. Both studies identified that practitioners used CAS primarily to have students visualize mathematical concepts and for students to explore and experiment with mathematical concepts. The literature review identified solving complex or real-world problems as the third most widely utilized purpose, rather than for homework and assignments, as reported in the international survey. Many issues with respect to CAS integration were discussed by mathematicians in the survey and echoed in the literature review, such as the cost of CAS, course time restrictions, and syntax. However, the issue of assessment was reported more frequently in the literature review than in Lavicza’s international survey. Potential benefits such as assisting with students’ future careers, aiding student comprehension, and making more complex mathematics accessible earlier, which are all results of Lavicza’s international survey, are also echoed in the literature review. The international survey examined the nature of mathematical literacy and its relationship to CAS with the literature review providing support for Lavicza’s findings such as an acceptance of CAS being similar to calculators, and technology being a necessity in the 21st century.

The literature review has contributed to four of the concluding implications from Lavicza’s research, whereas the first implication—to more holistically examine technology integration in post-secondary mathematics education—can also be seen as the guiding principle for this proposed comparative study. Indeed, bringing together these two types of different studies which deal with the same topic has shed light on certain significant areas of intersection (e.g., results somewhat ’independent’ of the methodology), and potential future direction for the two studies (i.e., based on complementary results justified by the different methodologies).

It is our intention, through future research, to continue to contribute to these implications, with the purpose of examining in-depth CAS use and sustainability at the post-secondary level. More specifically, our research program, in which the comparative study took place, aims at further addressing the second and third implications stemming from Lavicza’s international survey through documenting post-secondary teaching practices involving technology, and formulating recommendations for individual and departmental change. Further, in order to support Lavicza’s suggestion that collaboration between mathematicians and mathematics educators needs to be encouraged, our program aims at increasing the amount of attention being paid to post-secondary mathematics teaching from a research perspective and, more pragmatically speaking, elaborating on specific issues relating to, and strategies for, systemic integration of technology in post-secondary mathematics departments. This larger, ongoing study builds upon the results of the two studies mentioned above, through the implementation of an online survey regarding Canadian mathematicians’ teaching practices, two international case studies, the expansion of the literature review pilot study, and the organization of public workshops at Canadian research institutes which were specifically aimed at fostering discussions, between mathematicians and mathematics educators, on technology use in post-secondary mathematics instruction.

When the earliest papers from the literature review were written, floppy disks and dial-up internet were not anachronisms. Fifteen years later, today’s smart-phones not only have more computing power than the desktop computer of yesteryear but can also provide access to powerful clusters of servers on the internet that perform the functions of computer applications as well as manage a user’s data (i.e., “cloud computing”). A clear example of this phenomenon is Wolfram Alpha[7] which provides, among many other features, free access to a Mathematica-like environment over the web and via an iPhone application. While not a fully-featured CAS, it nevertheless can provide automatic solutions (not only answers, rather complete solutions with detailed justifications) to many traditional first-year mathematics and statistics questions through a syntax that supports a great deal of natural-language based queries. We submit that such technologies, combined with their inexpensive access and portability, have the potential to dramatically affect the post-secondary mathematics classroom, whether technology is formally integrated by instructors in the course or not. It may be that like the floppy disk, the question of whether or not to permit CAS use in post-secondary settings will become a relic of a bygone era, and that mathematicians and mathematics educators will need, at the very minimum, to investigate how these technologies are changing the post-secondary learning environment, with or without their direct involvement.

This research was funded by the Social Sciences and Humanities Research Council of Canada (SSHRC) as an International Opportunities Fund Project Grant (IOFDG), by internal grants received at Nipissing University and Brock University, and by Government of Ontario Work Experience grants. This manuscript summarizes Marshall's Honour's Thesis work (Department of Mathematics, Brock University, Canada; supervised by Buteau).

[7] See http://www.wolframalpha.com.

References

Alexander, M. (1996). College Algebra in an open classroom connecting teachers and technology. In: *Electronic proceedings of the ninth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL09/C024/paper.pdf. Accessed 12.11.09.

Alexander, M. (1998). Interactive computer software in college algebra. In: *Electronic proceedings of the eleventh annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL11/C001/paper.pdf. Accessed 12.11.09.

Artigue, M. (2002). Learning mathematics in a CAS environment: the genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*(3), 245–274.

Avioli, J. J. (1994). MapleCalc. In: *Electronic proceedings of the seventh annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL07/C016/paper.txt. Accessed 12.11.09.

Barton, S. (1995). Reluctant reformers' instructional practice and conceptions of teaching Calculus when using supercalculators. In: *Electronic proceedings of the eighth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL08/C007/paper.pdf. Accessed 12.11.09.

Berry, J. (1999). CAS as a mentor for the apprentice mathematician: Exploring CAS as a pedagogical vehicle towards expressiveness and explicitness in mathematics. In: *Proceedings of the CAME meeting at the Weizmann institute of science*. Retrieved November 12, 2009 from http://www.lkl.ac.uk/research/came/events/weizmann/CAME-Keynotes.pdf. 73–77. Accessed 12.11.09.

Buteau, C., Marshall, N., Jarvis, D. H., & Lavicza, Z. (2010). Integrating computer algebra systems in post-secondary mathematics education: preliminary results of a literature review. *International Journal for Technology in Mathematics Education, 17*(2), 56–68.

Challis, N. (2001). Expressive and explicit CAS: Is it enough? In: *Proceedings of CAME symposium*. http://www.lkl.ac.uk/research/came/events/freudenthal/4-Presentation-Challis.pdf. Accessed 12.11.09.

Cherkas, B. (2003). Finding polynomial and rational function turning points in pre-calculus. *International Journal of Computers for Mathematical Learning, 8*(2), 215–234.

Dana-Picard, T., & Steiner, J. (2003). Enhancing conceptual insight using a CAS. In: *Proceedings of CAME symposium*. http://www.lkl.ac.uk/research/came/events/reims/3-ShortPres-DanaPicard.pdf. Accessed 12.11.09.

Dogan-Dunlop, H. (2003). Technology-supported inquiry-based learning in Collegiate mathematics. In: *Electronic proceedings of the sixteenth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL16/C047/paper.pdf. Accessed 12.11.09.

Emese, G. (1994). Examples of graphing calculator use in Calculus I. In: *Electronic proceedings of the seventh annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL07/C019/paper.pdf. Accessed 12.11.09.

Herwaarden, O. & Gielen, J. (2001). Linking computer algebra systems and paper-and-pencil techniques to support the teaching of mathematics. *Proceedings of CAME symposium*. http://www.lkl.ac.uk/research/came/events/freudenthal/1-vHerwaarden.doc. Accessed 12.11.09.

Hohenwarter, M., Preiner J., & Yi, T. (2007). Incorporating GeoGebra into teaching mathematics at the college level. In: *Electronic proceedings of the nineteenth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL19/S100/paper.pdf. Accessed 12.11.09.

Hoyles, C., & Lagrange, J. B. (2010). *Mathematics education and technology —Rethinking the terrain: The 17th ICMI study*. New York: Springer.

Jarvis, D., Lavicza, Z., Buteau, C., & Marshall, N. (in press). Integrating CAS in post-secondary education - A literature review based on a multidimensional framework.

Keating, S. (2004)Restructuring Calculus III to integrate Maple technology in the teaching and learning process. In: *Electronic proceedings of the seventeenth annual international conference on technology in collegiate mathematics* http://archives.math.utk.edu/ICTCM/VOL17/C058/paper.pdf. Accessed 12.11.09.

Kramer, K. (1997). Using mathematica in a course on number theory. In: *Electronic proceedings of the tenth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL10/P009/paper.pdf. Accessed 12.11.09.

Kreczner, R. (1995). An interesting problem from Calculus and much more. In: *Electronic proceedings of the eighth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL08/C011/paper.pdf. Accessed 12.11.09.

Lagrange, J. B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and mathematics education: multidimensional overview of recent research and innovation. In F. K. S. Leung (Ed.), *Second international handbook of mathematics education, Vol. 1* (pp. 237–270). Dordrecht: Kluwer Academic Publishers.

Lavicza, Z. (2006). The examination of Computer Algebra Systems (CAS) integration into university-level mathematics teaching. In C. Hoyles, J. B. Lagrange, L. H. Son, & N. Sinclair (Eds.), *Online proceedings for the 17th ICMI study conference* (pp. 37–44). Hanoi, Vietnam: Hanoi University of Technology.

Lavicza, Z. (2008). The examination of Computer Algebra Systems (CAS) integration into university-level mathematics teaching. Unpublished Ph.D. dissertation, The University of Cambridge, Cambridge, UK.

Little, R. E. (1994). Using a calculator to do mathematics: Do you want a map or just directions? In: *Electronic proceedings of the seventh annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL07/C005/paper.html. Accessed 12.11.09.

May, M. (1999). Designing courseware with Maple. In: *Electronic proceedings of the twelfth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL12/P013/paper.pdf. Accessed 12.11.09.

Pierce, R. L., & Stacey, K. (2004). A framework for monitoring progress and planning teaching towards the effective use of computer algebra systems. *International Journal of Computers for Mathematical Learning, 9*(1), 59–93.

Pollack-Johnson, B., & Borchardt, A. (1995). Using the TI-82 and spreadsheets in a 1st-year math course for business and social science. In: *Electronic proceedings of the eighth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL08/C045/paper.txt. Accessed 12.11.09.

Power, B. L. (2000). Exploring Optimization in Calculus-Student Projects in Finding Absolute Extrema on a Closed Interval Domain. Electronic Proceedings of the Thirteenth Annual International Conference on Technology in Collegiate Mathematics. http://archives.math.utk.edu/ICTCM/VOL13/P003/paper.pdf. Accessed 12.11.09.

Putz, J. F. (1995). The CAS in multivariable calculus. In: *Electronic proceedings of the eighth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL08/C016/paper.html. Accessed 12.11.09.

Savari, C. (2005). Experiences in the integration of CAS into engineering education at the University of Pecs. In: *Proceedings of CAME symposium*. http://www.lkl.ac.uk/research/came/events/CAME4/CAME4-topic2-ShortPaper-Sarvari.pdf. Accessed 12.11.09.

Schlatter, M. D. (1999). Using Matlab in a Multivariable Calculus Course. Electronic Proceedings of the Twelfth Annual International Conference on Technology in Collegiate Mathematics. http://archives.math.utk.edu/ICTCM/VOL12/C021/paper.pdf. Accessed 12.11.09.

Son, L., Sinclair, J., Lagrange, J.B., & Hoyles, C. (Eds.). (2006). *Proceedings of the ICMI 17 study conference: digital technologies and mathematics teaching and learning: rethinking the terrain*. 3rd–8th December, 2006, Hanoi (Vietnam) [CD-ROM].

Spitznagel, C. R. (1994). Twelve practical ideas for the use of the TI-82 in the second-term calculus course. In: *Electronic proceedings of the seventh annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL07/C020/paper.pdf. Accessed 12.11.09.

Tunis, H. (2004). Letter to the editor. *Educational Studies in Mathematics, 57*(1), 145–146.

Turegun, M. (2006). Computing critical test statistic values using tables! Tables?!.What tables?! Use your TI-83/84. In: *Electronic proceedings of the eighteenth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL18/C015/paper.pdf. Accessed 12.11.09.

White, J. (1997). Retention and attitudes toward mathematics and computers: Their relationship with using computers in introductory college mathematics courses. In: *Electronic proceedings of the tenth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL10/C015/paper.pdf. Accessed 12.11.09.

Wilson, W. S. (2005). Short response to Tunis's 'Letter to the Editor' on technology in college. *Educational Studies in Mathematics, 58*(3), 415–420.

Wilson, W., & Naiman, D. (2004). K-12 calculator usage and college grades. *Educational Studies in Mathematics, 56*(1), 119–122.

Wrangler, T. (1995). Paradigm lost: A modern approach to teaching ordinary differential equations. In: *Electronic proceedings of the eighth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL08/C059/paper.pdf. Accessed 12.11.09.

Wu, D. (1995). Some examples on using Maple to increase students' understanding of calculus. In: *Electronic proceedings of the eighth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL08/C052/paper.pdf. Accessed 12.11.09.

Xie, S. (1994). Interactive laboratories for teaching Applied Mathematics. In: *Electronic proceedings of the seventh annual international conference on technology in collegiate mathematics*. Retrieved November 12, 2009 from http://archives.math.utk.edu/ICTCM/VOL07/C032/paper.pdf. Accessed 12.11.09.

Xu, M. (1995). Using graphing calculators to motivate students in a calculus course. In: *Electronic proceedings of the eighth annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL08/C037/paper.pdf. Accessed 12.11.09.

Yates, K. (1994). Meshing meaningful student projects into a fairly traditional calculus course. In: *Electronic proceedings of the seventh annual international conference on technology in collegiate mathematics*. http://archives.math.utk.edu/ICTCM/VOL07/C018/paper.pdf. Accessed 12.11.09.