Postsecondary mathematics education and technology: some personal views from a mathematician's perspective

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Fields Institute
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UNIVERSITÉ LAVAL
Faculté des sciences et de génie

PLAN OF THE TALK

I- Mathematicians and mathematics education research

II- The influence of computers and informatics on mathematics and its teaching: a brief historical survey

III- Technology in postsecondary mathematics education nowadays

I- Mathematicians and mathematics education research

Research mathematician

Long tradition of a “serious” involvement of some mathematicians

- in general pedagogical issues
- in the education of school teachers

International Commission on Mathematical Instruction

(Established at the Fourth International Congress of Mathematicians, Rome, 1908)

First international association concerned with the teaching of a discipline

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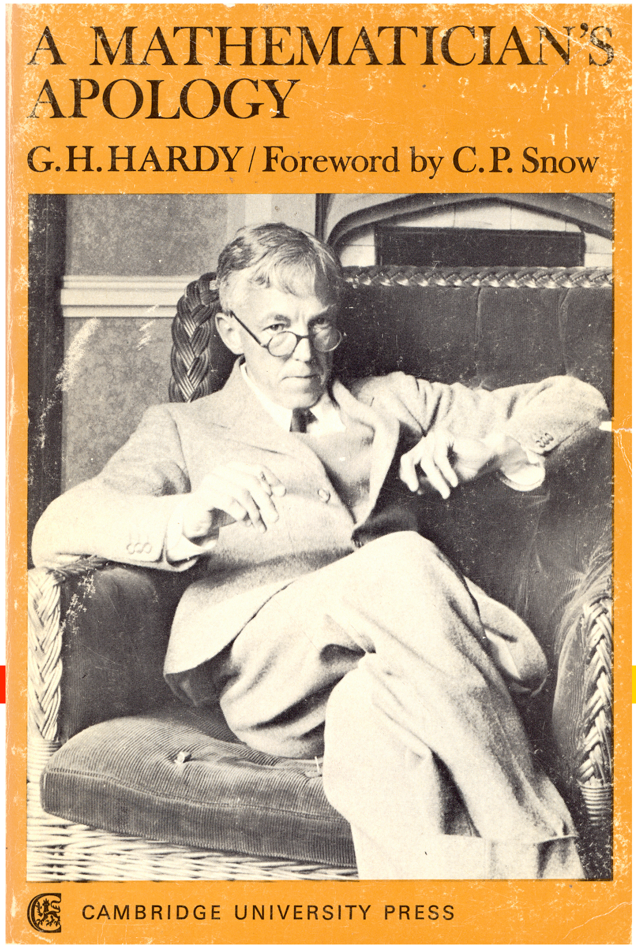
Two myths

- "First, it is a common belief among ***mathematicians*** that attention to education is a kind of pasturage for mathematicians in scientific decline."
  (as is attention to history of maths)



cf G.H. Hardy, *A Mathematician's Apology*

Hyman Bass (BAMS 2005) "Mathematics, mathematicians and mathematics education"

Two myths

- "First, it is a common belief among ***mathematicians*** that attention to education is a kind of pasturage for mathematicians in scientific decline."
  (as is attention to history of maths)

"My examples include scholars of substantial stature in our profession and in highly productive stages of their mathematical careers."

Hyman Bass (BAMS 2005) "Mathematics, mathematicians and mathematics education"

Two myths *(continued)*

- "Second, many ***educators*** have questioned the relevance of contributions made by research mathematicians."

"I will argue that the knowledge, practices, and habits of mind of research mathematicians are not only relevant to school mathematics education, but that this mathematical sensibility and perspective is essential for maintaining the mathematical balance and integrity of the educational process—in curriculum development, teacher education, assessment, etc."

Hyman Bass (BAMS 2005) "Mathematics, mathematicians and mathematics education"

Five eloquent examples

(1854 – 1912)

Henri Poincaré

vg articles in
L'Enseignement Mathématique
(journal established in Geneva in 1899)

- *notations*
  (links between the differential notation and teaching, 1899)

- *definitions*
  (role of definitions in mathematics, 1904)

(1887 – 1985)

George Pólya

How to solve it? (1945)

mathematical knowledge:

- *information*
- *know-how*

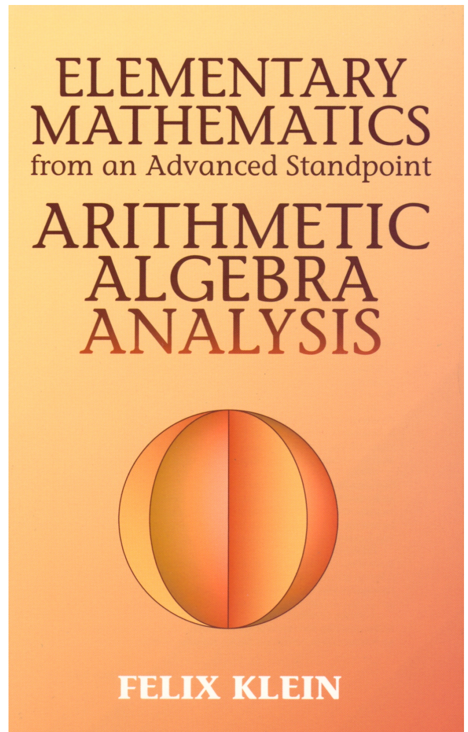
"Know-how is much more important than the mere possession of information."
(Pólya, *Mathematical Discovery*)

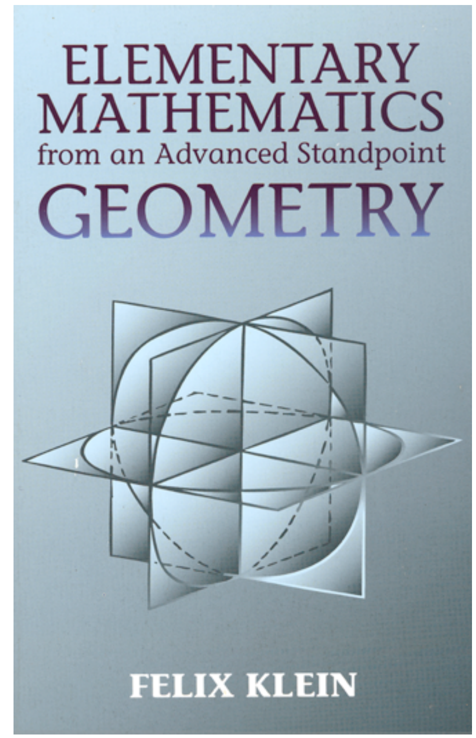
"*Pedagogical Content Knowledge*"
(Shulman, 1986)

"*Mathematical Knowledge for Teaching*"
(Ball/Bass, 2003)

Three presidents of ICMI

Elementarmathematik vom höheren Standpunkte aus (1908)





Felix Klein

(1849 – 1925)

(First ICMI president, 1908 – 1920)

Distinguished mathematician

(Erlangen Programme)

International Commission on Mathematical Instruction

The Klein Example

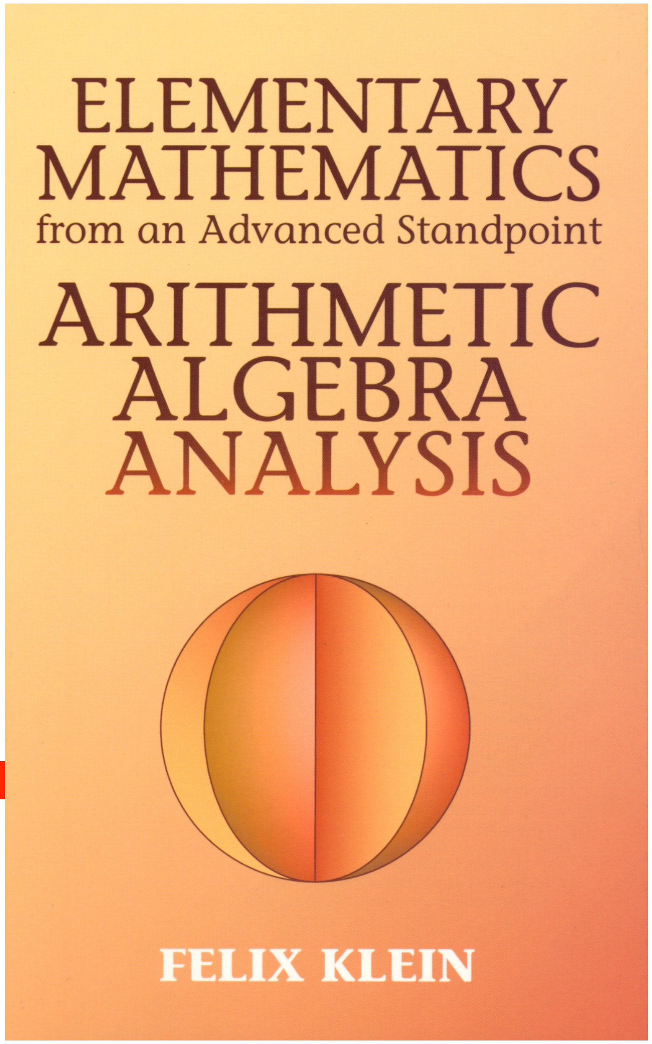
- An accomplished and broadly cultured mathematician
- An engaged interest in mathematics teaching and learning
- A sensitive respect for teachers and the professional demands of their work
- A true open-mindedness toward educators

"I believed that the whole sector of Mathematics teaching, from its very beginnings at elementary school right through to the most advanced level research, should be organised as an organic whole. It grew ever clearer to me that, without this general perspective, even the purest scientific research would suffer, inasmuch as, by alienating itself from the various and lively cultural developments going on, it would be condemned to the dryness which afflicts a plant shut up in a cellar without sunlight."

[Felix Klein 1923]

Preface:

"I shall endeavor to put before the teacher, as well as the maturing student, from the view-point of modern science, but in a manner as simple, stimulating, and convincing as possible, both the content and the foundations of the topics of instruction, with due regard for the current methods of teaching."



Aim:
to mend the ***double discontinuity*** that exists between secondary education and higher education in mathematics

„doppelte Diskontinuität"

"Proof without words" of the formula

$$(a - b)(c - d) = ac - ad - bc + bd$$

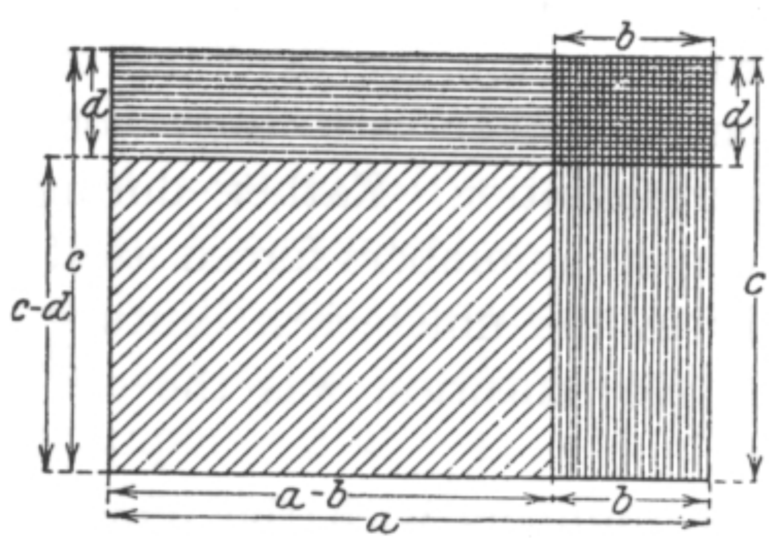


Fig. 6.

But even more interesting as regards today's workshop…

machine only if you examine it afterwards personally and if you see, by actual use, how it is operated. The machine will be at your disposal, for that purpose, after the lecture.

So far as the *external appearance* of the Brunsviga is concerned, it presents schematically a picture somewhat as follows (see Fig. 1, p. 18). There is a fixed frame, the "*drum*", below which and sliding on it, is a smaller longish case, the "*slide*". A handle which projects from the drum on the right, is operated by hand. On the drum there is a series of parallel slits, each of which carries the digits $0, 1, 2, \ldots, 9$, read downwards; a peg s projects from each slit and can be set at pleasure at any one of the ten digits. Corresponding to each of these slits there is an opening on the slide under which a digit can appear. Figure 3, p. 19 gives a view of a newer model of the machine.

I think that the arrangement of the machine will be clearer if I describe to you the process of carrying out a definite calculation, and the way in which the machine brings it about. For this I select *multiplication*.

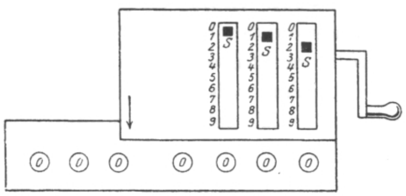


Fig. 1. Before the first turn.

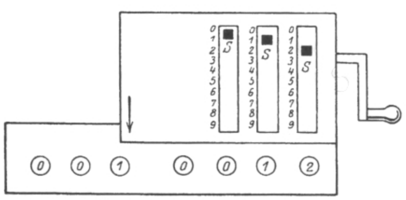


Fig. 2. After the first turn.

The procedure is as follows: One *first sets the drum pegs on the multiplicand*, i. e., beginning at the right, one puts the first lever at the one's digit, the second at the ten's digit of the multiplicand, etc. If, for example, the multiplicand is 12, one sets the first lever at 2, the second lever at 1; all the other levers remain at zero (see Fig. 1). *Now turn the handle once around, clockwise*. The multiplicand appears under the openings of the slide, in our case a 2 in the first opening from the right, a 1 in the second, while zeros remain in all the others. Simultaneously, however, in the first of a series of openings in the slide, at the left, the digit 1 appears to indicate that we have turned the handle once (Fig. 2). *If now one has to do with a multiplier of one digit, one turns the handle as many times as this digit indicates; the multiplier will then be exhibited on the slide to the left, while the product will appear on the slide to the right*. How does the apparatus bring this result about? In the first place there is attached to the under side of the slide, at the left, a cogwheel which carries, equally spaced on its rim, the digits $0, 1, 2, \ldots, 9$. By means of a driver, this cogwheel is rotated through one tenth of its perimeter with every turn of the handle, so that a digit becomes visible through the opening in the slide, which actually indicates

the number of revolutions, in other words the multiplier. Now as to the *obtaining of the product*, it is brought about by similar cogwheels, one under each opening at the right of the slide. But how is it that by one and the same turning of the handle, one of these wheels, in the above case, moves by one unit, the other by two? This is where the peculiarity in construction of the Brunsviga appears. Under each slit of the drum there is a flat wheel-shaped disc (driver) attached to the axle of the handle, upon which there are nine teeth which are movable in a radial direction (see Fig. 4). By means of the projecting peg S, mentioned above, one can turn a ring R which rests upon the periphery of the disc, so that, according to the mark upon which one sets S in the slit, $0, 1, 2, \ldots, 9$ of the movable teeth spring outward (in Fig. 4, two teeth). These teeth engage the cogs under the corresponding openings of the slide, so that *with one turn of the handle each driver thrusts forward the corresponding cogwheel by as many units as there are teeth pushed out*, i. e., *by as many teeth as one has set with the corresponding peg S*. Accordingly, in the above illustration, when we start at the zero position, and turn the handle once, the units wheel must jump to 2, the ten's wheel to 1, so that 12 appears. A second turn of the handle moves the units wheel another 2 and the tens wheel another 1, so that 24 appears, and similarly, we get, after 3 or 4 times, $3 \cdot 12 = 36$ or $4 \cdot 12 = 48$, respectively.

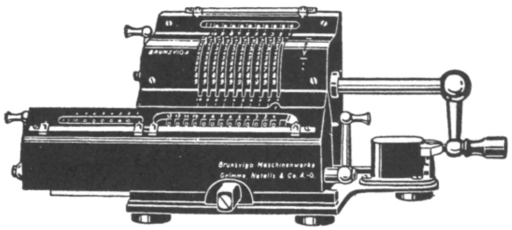


Fig. 3.

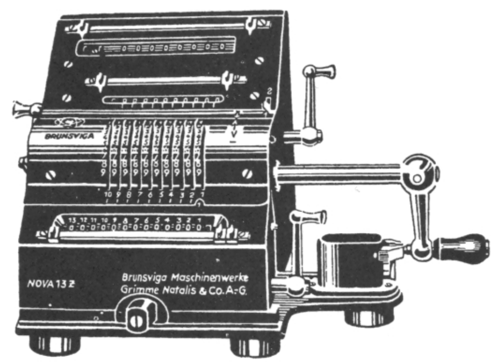


Fig. 3a.

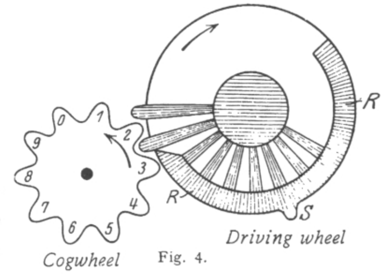
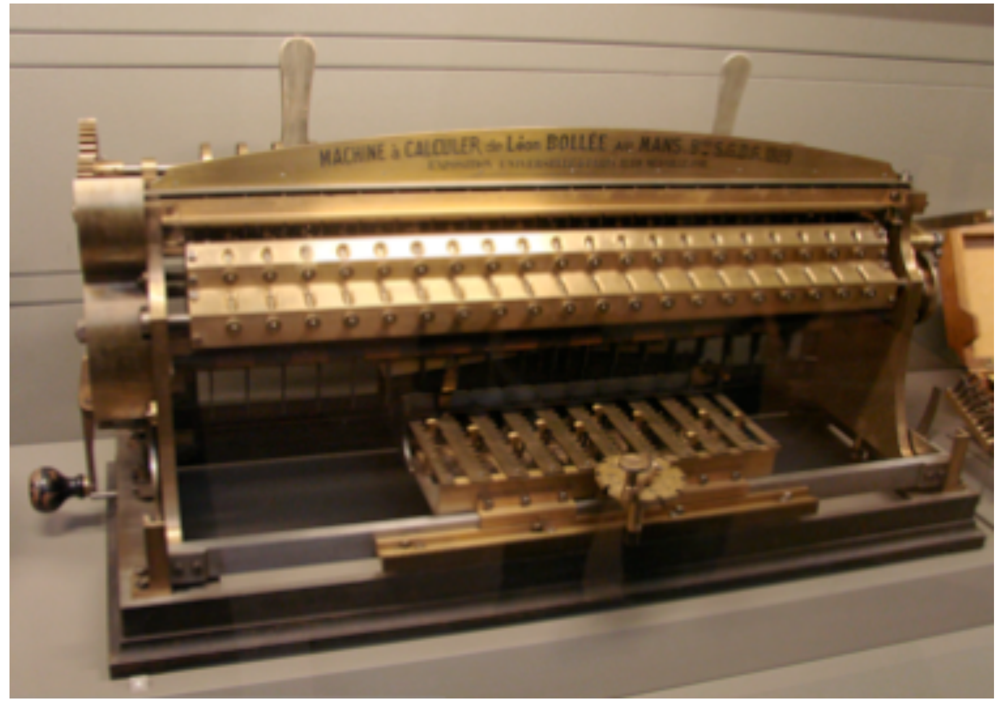
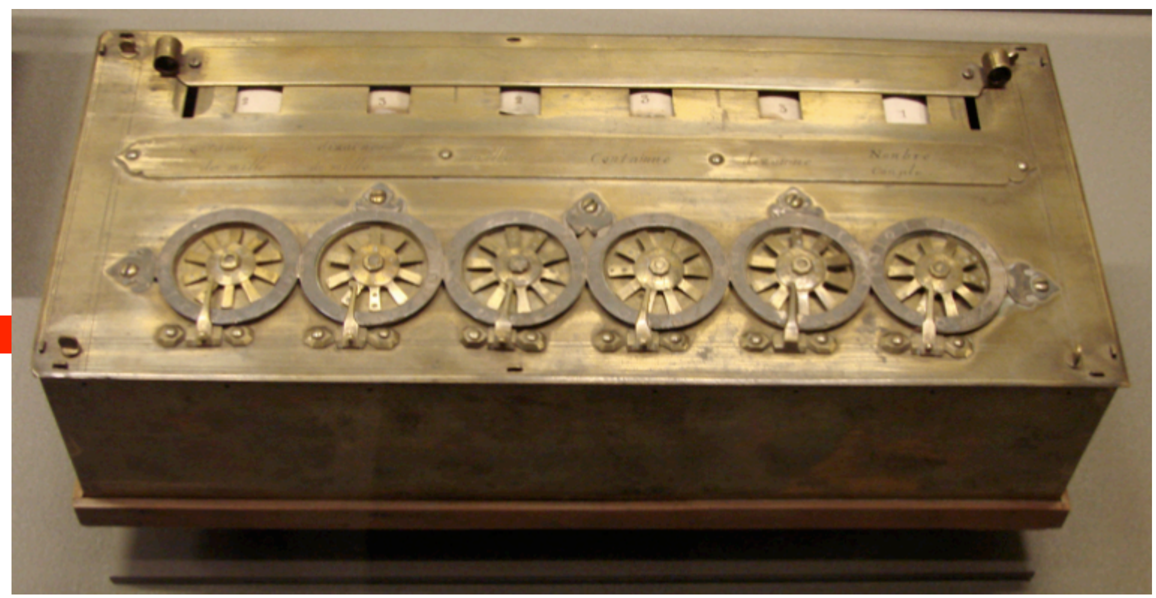


Fig. 4.

Léon Bollée's calculating machine (1888)



Pascaline (1642)



Hans Freudenthal
(1905 – 1990)

(Eight ICMI president, 1967 – 1970)

- Important career as a research mathematician
  (topology, Lie groups, logic, prob & stat)
- Founded the journal *Educational Studies in Mathematics (1968)*
- Instigated the ICME congresses
  (International Congress on Mathematical Education – 1969)
- Created a major research institute in mathematics education
  (has become the FIsme, Utrecht)

"Un homme d'action"
(J. Adda, *ESM* 1993)

International Commission on Mathematical Instruction

The rationale behind such initiatives:

"The theory of mathematical education is becoming a science in its own right, with its own problems both of mathematical and pedagogical content. The new science should be given a place in the mathematical departments of Universities or Research Institutes, with appropriate academic qualifications available."

Resolution adopted at ICME-1 (1969)

The emergence of math education as a scientific discipline (“didactics of mathematics”)

Hyman Bass
(1932 –)

(Fourteenth ICMI president, 1999 – 2006)

- Outstanding career as a research mathematician (Columbia Univ.) *(homological algebra, algebraic K-theory – notorious "Bourbaki")*
- President of the Mathematical Sciences Education Board (1992-2000) – US National Academy of Sciences
- President, AMS (2001-2002)
- Still active in math education research, in particular about primary school teacher education
  "Mathematical Knowledge for Teaching"
  (MKT – D. Ball & H. Bass, 2003)

Mathematicians involved
- in postsecondary education
- in primary and secondary education

International Commission on Mathematical Instruction

Fruitful collective involvements

Canadian Mathematical Society

- "Education" pages of the CMS website
- Education Prizes
    - Adrien-Pouliot Award
    - Excellence in Teaching Award(s) (2?)
- Education Committee
- Canadian Mathematics Education Forum
  (1995, 2003, 2005, 2009)
- support (financial and logistic)
  to ICME-7 (Québec, 1992)

ICME 7
QUÉBEC
1 9 9 2

Fruitful collective involvements (*continued*)

- support to ICME-7 (Québec, 1992)
- website

  Programs > Supporting Educational Initiatives
- papers in the *Notices*
- new education column "*Doceamus*" in the *Notices*

Fruitful collective involvements *(continued)*

MAA MATHEMATICAL ASSOCIATION OF AMERICA

Special Interest Group of the MAA

RUMEonline!

Special Interest Group of the MAA on
Research in Undergraduate Mathematics Education

- publications dans la série *MAA Notes*
- série *Research in Collegiate Mathematics Education*

ICME 7
QUÉBEC
1 9 9 2

Situations sometimes difficult

"Math Wars" episode (USA, 1990s)

(primary and secondary school math)

- not a glorious episode for the research mathematician!
- comments by A. Ralston

    (surely not an opponent of mathematics…)

"In the Math Wars the research mathematics community has departed from its own high intellectual standards for research and has displayed an arrogance that has made things much worse than they need have been. Of course, neither of these strictures applies to every research mathematician who has been involved with the Math Wars, but it applies to too many and particularly to many of those who have been most vocal."

Anthony Ralston (*Notices of the AMS*, 2004)
"Research mathematicians and mathematics education: a critique"

Comments by A. Ralston, *Notices* 2004 *(continued)*

"My conclusion is that although a number of research mathematicians have contributed positively to school mathematics education in recent years (…), the research mathematics community has largely squandered an opportunity to have a significant positive impact on American mathematics education. Too many have used a 'scattershot approach' that often takes the form of 'unsubstantiated claims and random anecdotes'. Too often the result has been that when they have become active in mathematics education, research mathematicians have not lived up to the high standards that they normally bring to their own professional work."

Comments by A. Ralston, *Notices* 2004 *(continued)*

"The most important way [to make a more positive contribution] would be for research mathematicians to see their role as colleagues of mathematics educators and *constructive* critics of work in mathematics education."

Comments on the role of technology in primary and secondary education
(calculators)

Situations sometimes difficult   *(continued)*

Tensions between ICMI and IMU

International Congress of Mathematicians – Berlin (1998)
organisation of the sessions in the section on
Teaching and popularisation of mathematics

International
Mathematical
Union (IMU)

Building a stronger collaboration between the communities of mathematicians and mathematics educators

International Commission on
Mathematical Instruction

Collaboration – mathematicians and math educators

Challenges for ICMI

- *at the institutional level:*
  maintain strong links with IMU as a body
  (institutionalise such links)
- *at the individual level:*
  attract more research mathematicians to ICMI activities
  (ICME congresses, Studies, etc. -- ***Study 20, with ICIAM***)

Analogous challenges on the Canadian scene

Canadian Mathematical Society

Canadian Mathematics Education Study Group

CMESG
GCEDM

At stake: mutual understanding and respect among mathematicians and math educators

As a conclusion to the first part

Yves Chevallard

Hans Freudenthal Award 2009

The integration of technology in postsecondary teaching as a framework for interaction and collaboration between mathematicians and math educators

PLAN OF THE TALK

I- Mathematicians and mathematics education research

II- The influence of computers and informatics on mathematics and its teaching: a brief historical survey

III- Technology in postsecondary mathematics education nowadays

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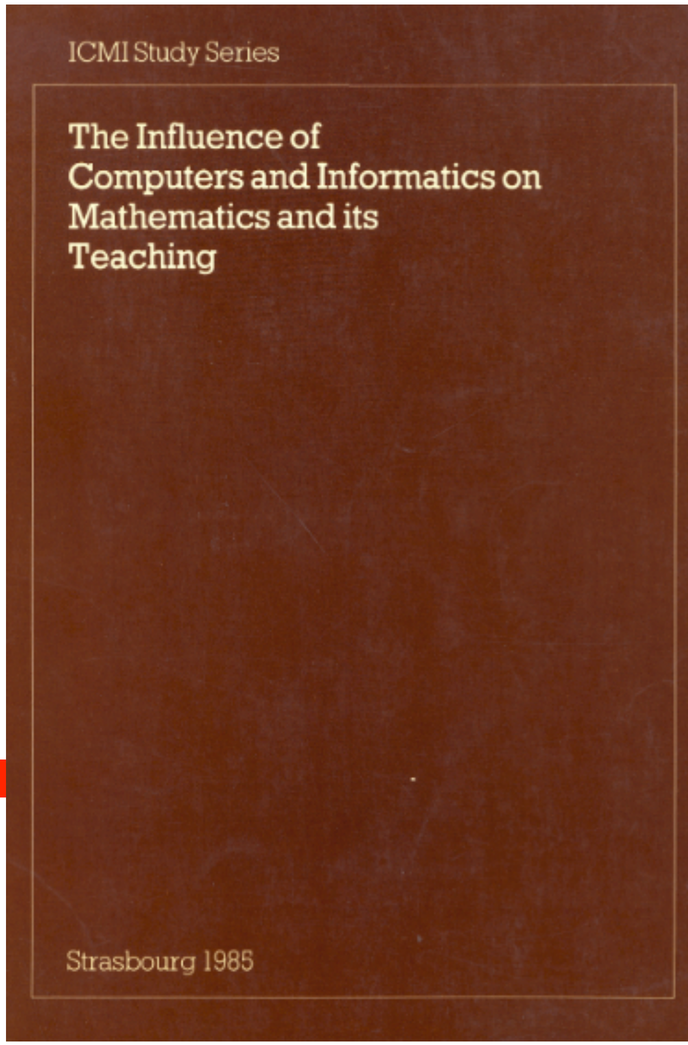
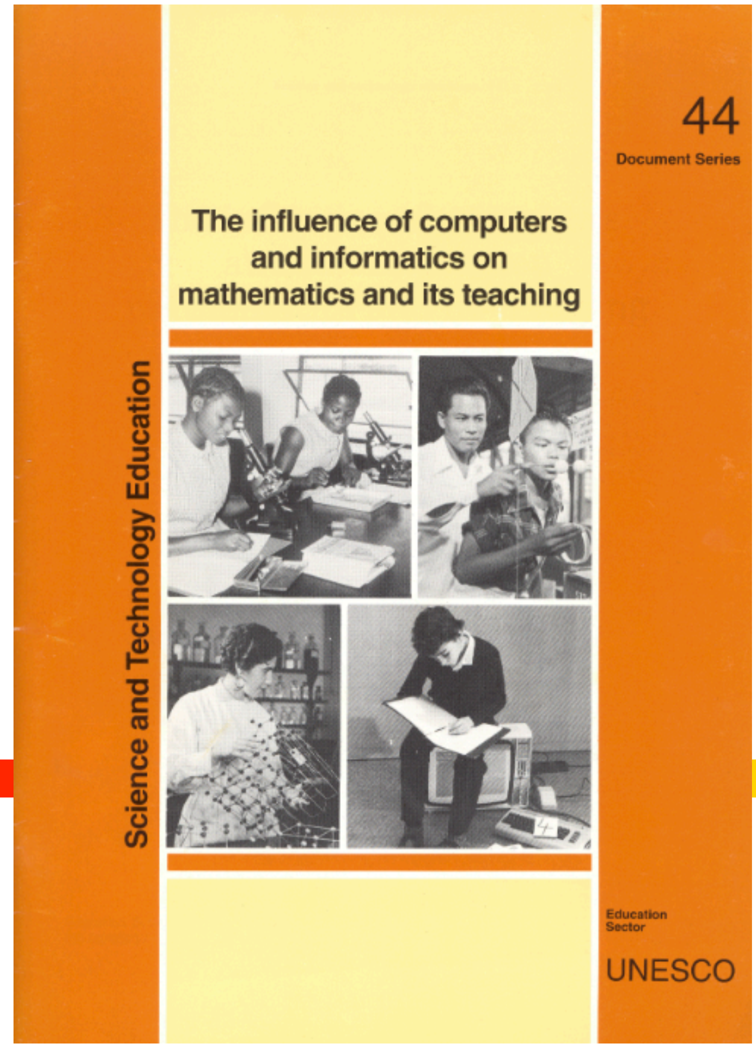
Faculté des sciences et de génie

II- The influence of computers and informatics on mathematics and its teaching: a brief historical survey





Some chronological milestones

(general and personal)

- ***1968*** MACSYMA
  Risch algorithm for symbolic integration - TAMS
- ***1969*** my first contact with APL at Univ. Laval
- ***1972*** … and with a handheld calculator!
- ***1978*** muMATH (my first contact -- 1981: Apple II)
- ***1980*** Maple
- ***1981*** my first contact with Logo

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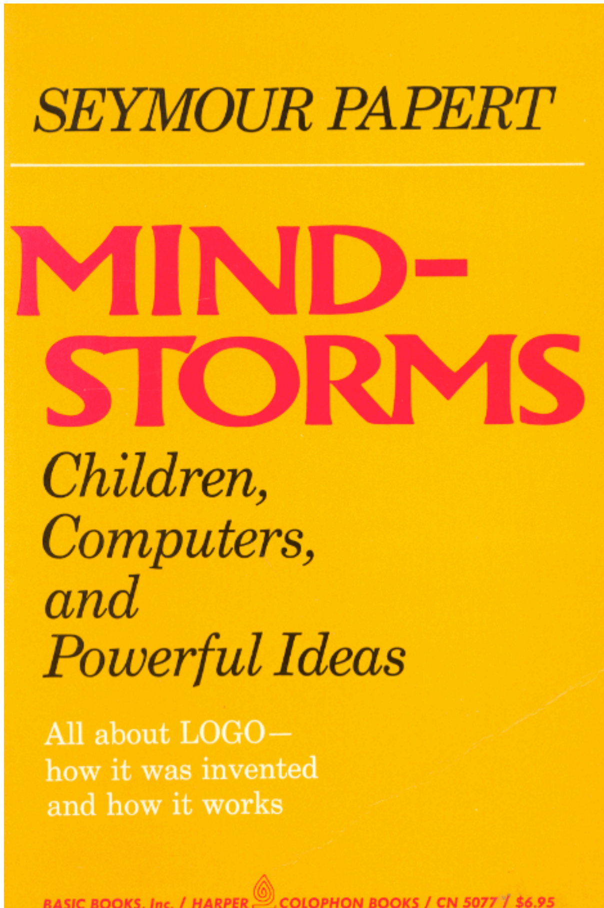
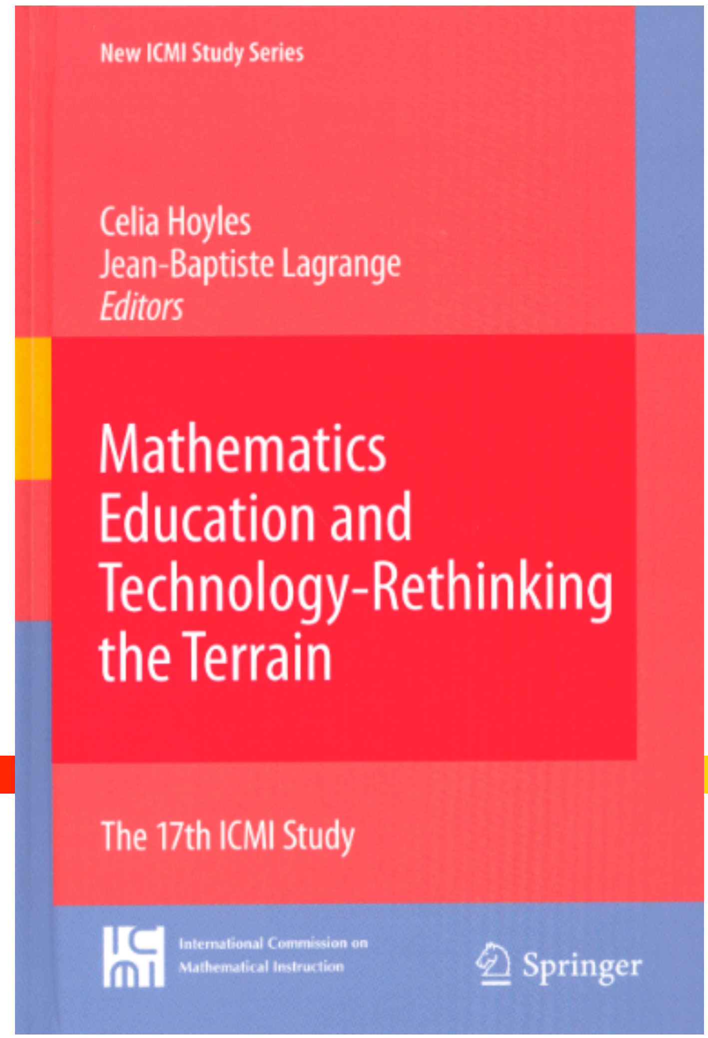


(1928 –)

Hanoi, December 2006



Revisiting the 1st ICMI Study

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THE DISK WITH THE COLLEGE EDUCATION

HERBERT S. WILF
Department of Mathematics, University of Pennsylvania, Philadelphia PA 19104

The title is somewhat exaggerated, but the calculators-or-no-calculators dilemma that haunts the teaching of elementary school mathematics is heading in the direction of college mathematics, and this article is intended as a distant early-warning signal.

I have in my home a small personal computer. About 500,000 small personal computers have been sold in this country, of which a healthy fraction are owned by individuals. I use mine primarily for word processing (this article was written on it), for writing programs that do various mathematical jobs related to my teaching or to my research, for playing games, for keeping class rolls, etc.

A new program has recently been made available for my little computer, one whose talents seem worthy of comment here because it knows calculus; in fact, as you read these words, some of

- ***1982*** Herbert S. Wilf: "The disk with the college education" (*Amer. Math. Monthly)*

your students may be doing their homework with it.

The program is called muMATH; it was written by the Soft Warehouse, and is distributed in the United States by Microsoft Consumer Products of Bellevue, Washington. It costs about $75 and is supplied on a 5-inch floppy disk with an (inadequate) instruction manual.

The program on the disk does numerical calculation to high precision, or symbolic manipulation of expressions. The numerical calculation, which is less important as far as this article is concerned, is in rational arithmetic and is done with 611-digit accuracy. Thus, for example, when the program is loaded, the question

?30!;

yields the instant answer

@265252859812191058636308480000000

The question

$$?1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7;$$

elicits

@363/140

and so forth.

- ***1982*** Herbert S. Wilf: "The disk with the college education" (*Amer. Math. Monthly)*

The disk, however, has graduated from high school. Here it is in a freshman calculus course. To differentiate $x \sin x$ with respect to x just ask

?DIF(X*SIN(X),X);

to obtain

@X*COS(X) + SIN(X)

At the risk of some eyestrain, we might even ask it to

?DIF((X ↑ 3 + COS(X)) ↑ (1/2),X);

to which it replies

(3*X ↑ 2*(X ↑ 3 + COS(X)) ↑ (1/2)/2 − (X ↑ 3 + COS(X) ↑ (1/2)*SIN(X)/2)/ (X ↑ 3 + COS(X))

DEPARTEMENT DE MATHEMATIQUES
CAFE DU LUNDI

MICHEL FORTIN
BERNARD R. HODGSON
ET COLLABORATEURS

SUJET: WHO NEEDS BRAINS
IF YOU'VE GOT
BATTERIES.

LUNDI 24 SEPTEMBRE
15:30
SALLE 2840

BIENVENUE A TOUS!

(1984)

Some chronological milestones

(general and personal)

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  Risch algorithm for symbolic integration - TAMS
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- ***1978*** muMATH (my first contact -- 1981: Apple II)
- ***1980*** Maple
- ***1981*** my first contact with Logo
- ***1982*** Herbert S. Wilf: "The disk with the college education" (*Amer. Math. Monthly)*
- ***1985*** 1st ICMI Study
  Cabri-géomètre
- ***1987*** HP-28C
  Wilf: "The chip with the college education: the HP-28C" (GCEDM/CMESG)

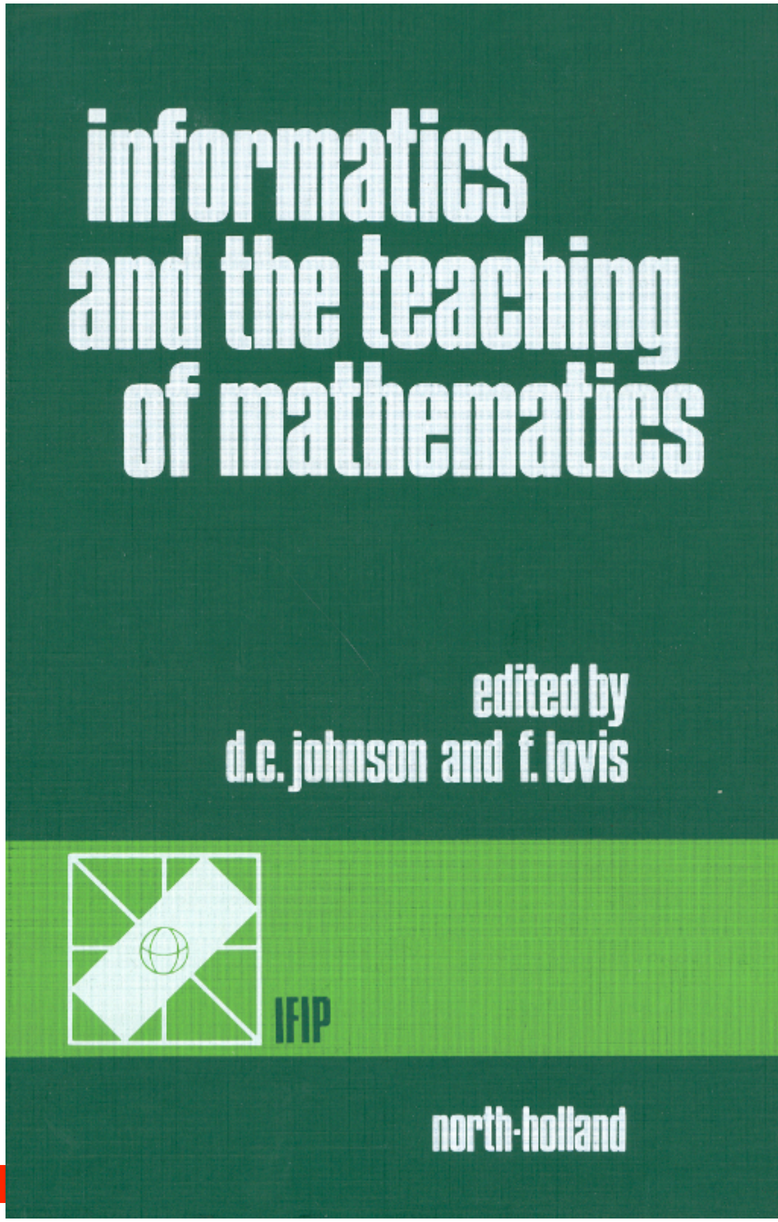
SCIENT. AMER. (FÉV. 87)

Reflections and analyses

an international movement

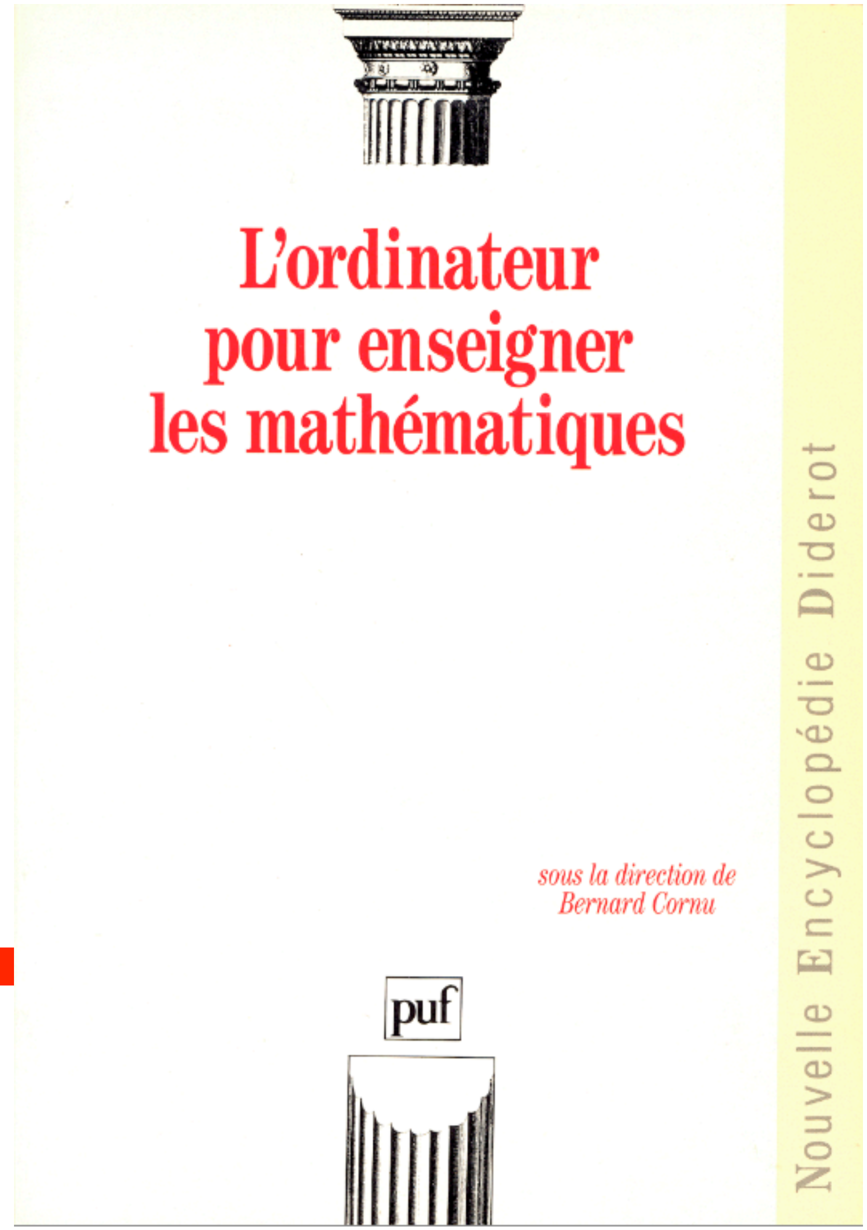
For instance in Canada: Working Groups at meetings of GCEDM / CMESG

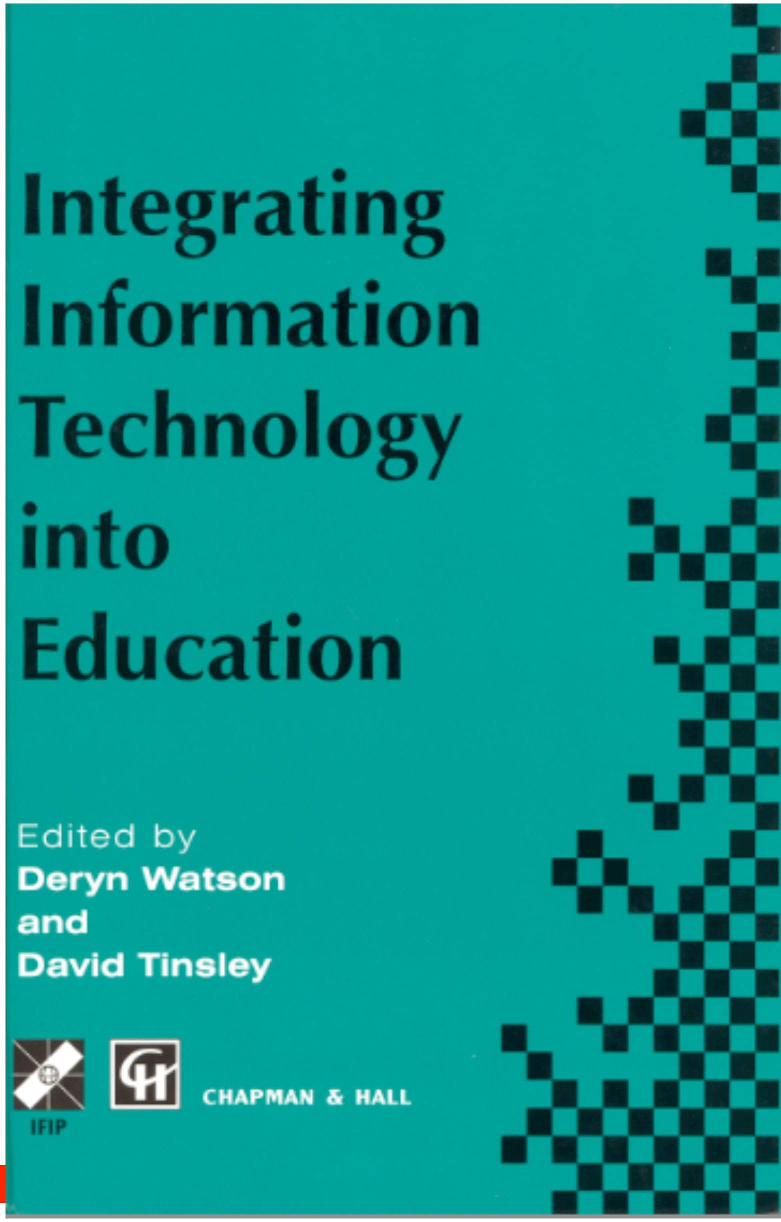
- ***1982*** The influence of computer science on undergraduate mathematics education
- ***1984*** Logo and the mathematics curriculum
- ***1985*** Impact of symbolic manipulation software on the teaching of calculus
- ***1989*** Computers in the undergraduate math curriculum
- ***1998*** Mathematical software for the undergraduate curriculum


informatics
and the teaching
of mathematics
edited by
d.c. johnson and f. lovis
IFIP
north-holland

1987

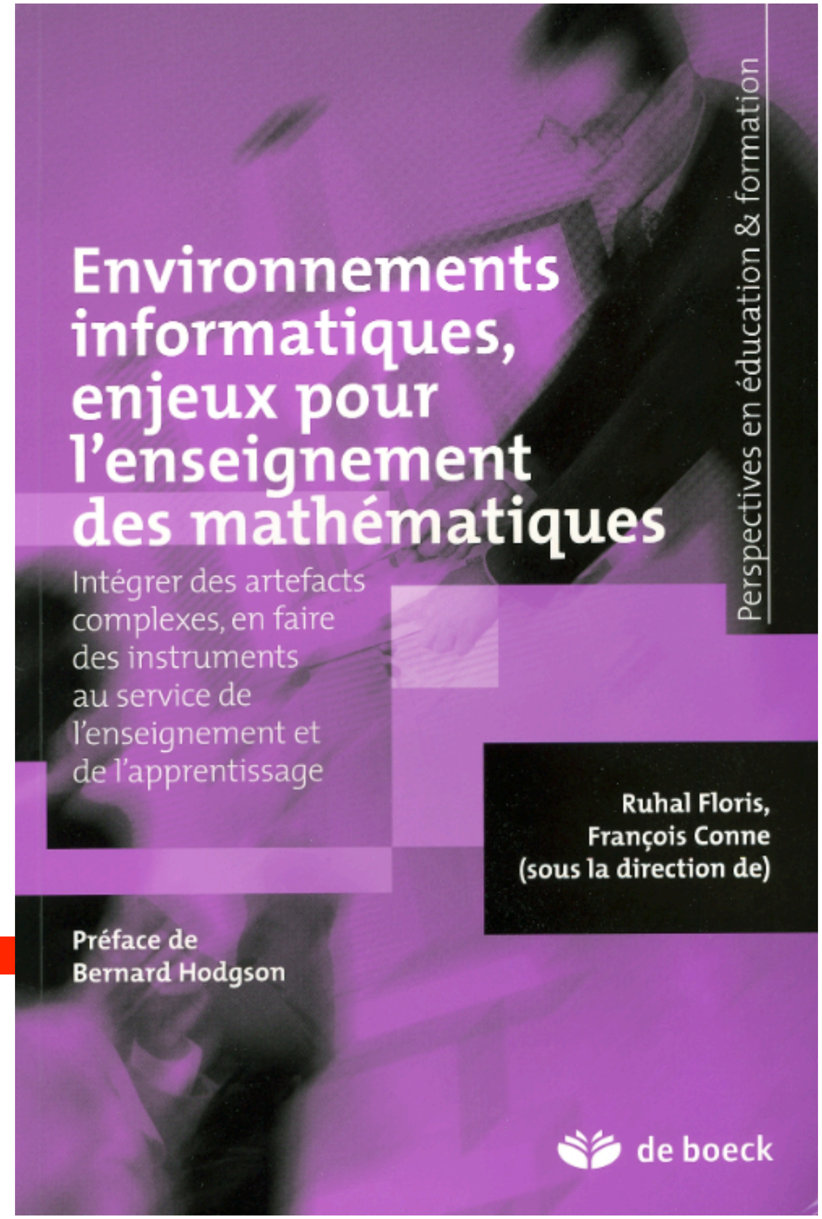
1992


L'ordinateur
pour enseigner
les mathématiques
sous la direction de
Bernard Cornu
puf
Nouvelle Encyclopédie Diderot

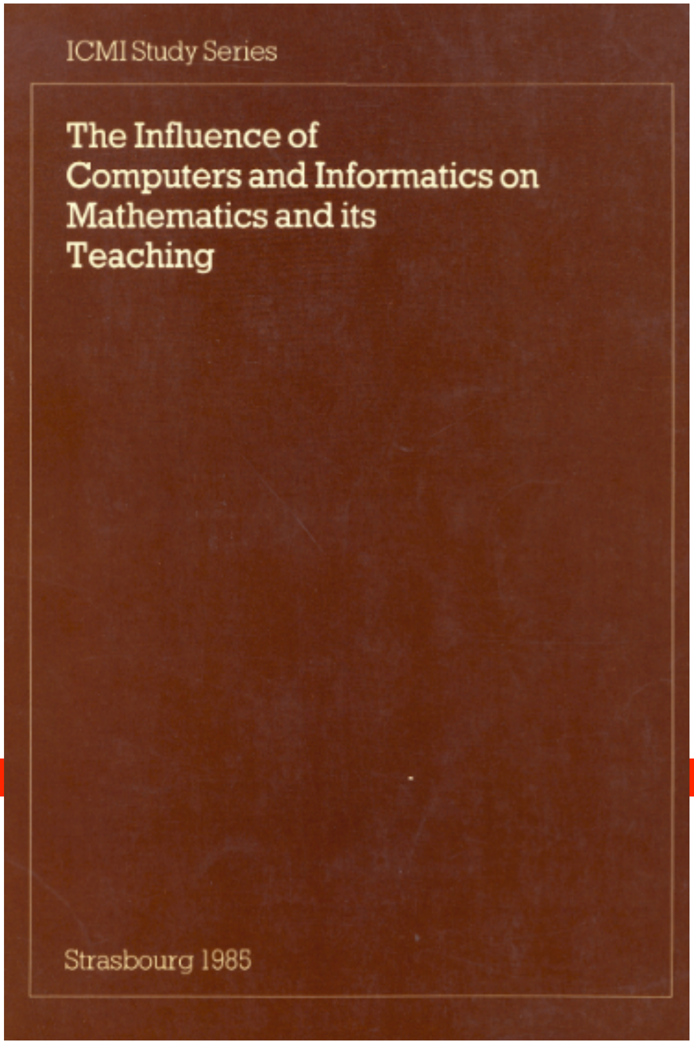

Integrating Information Technology into Education
Edited by
Deryn Watson
and
David Tinsley
IFIP
CHAPMAN & HALL

1994

2007


Environnements informatiques, enjeux pour l'enseignement des mathématiques
Intégrer des artefacts complexes, en faire des instruments au service de l'enseignement et de l'apprentissage
Ruhal Floris,
François Conne
(sous la direction de)
Préface de
Bernard Hodgson
de boeck
Perspectives en éducation & formation

The influence of *computers* and *informatics* on *mathematics* and its *teaching*



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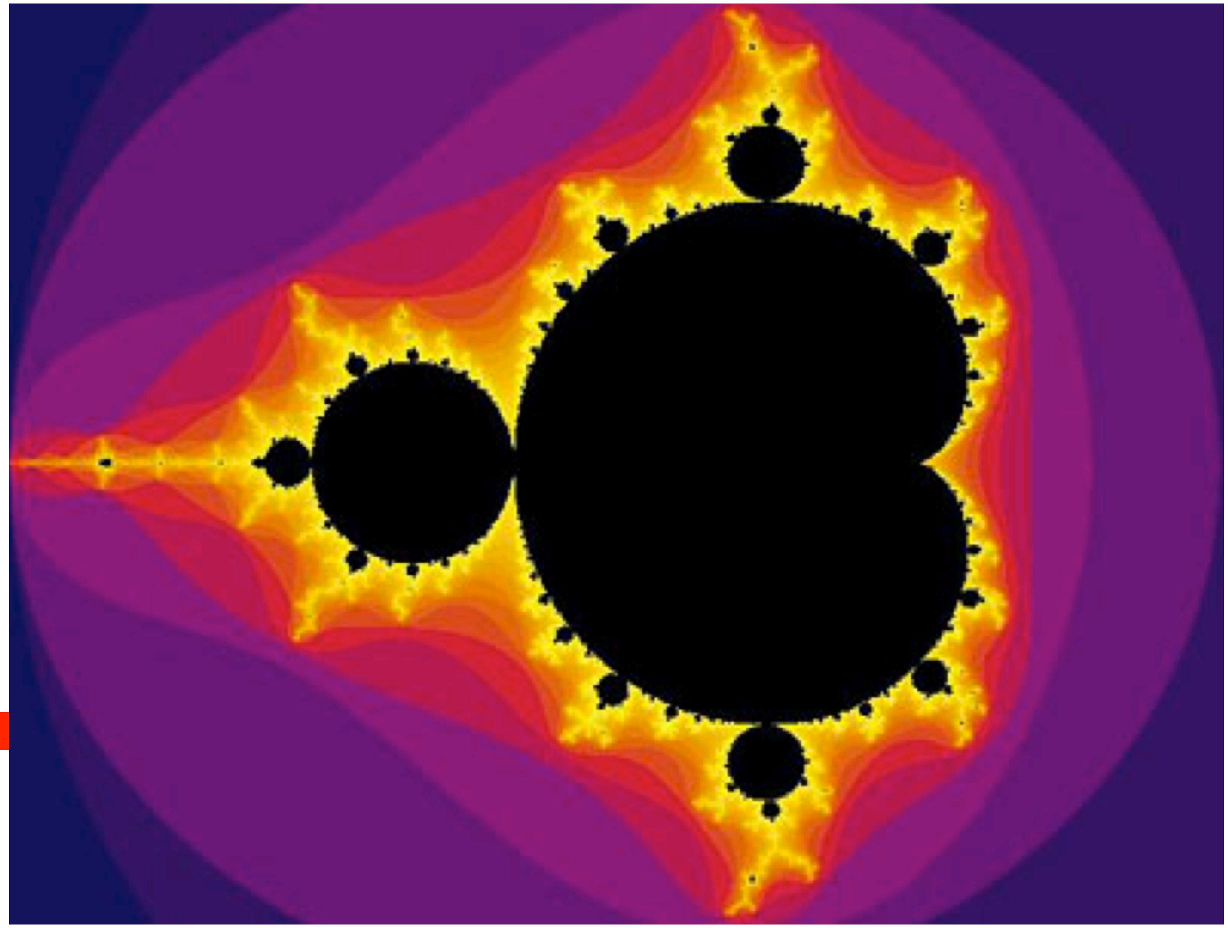
Faculté des sciences et de génie

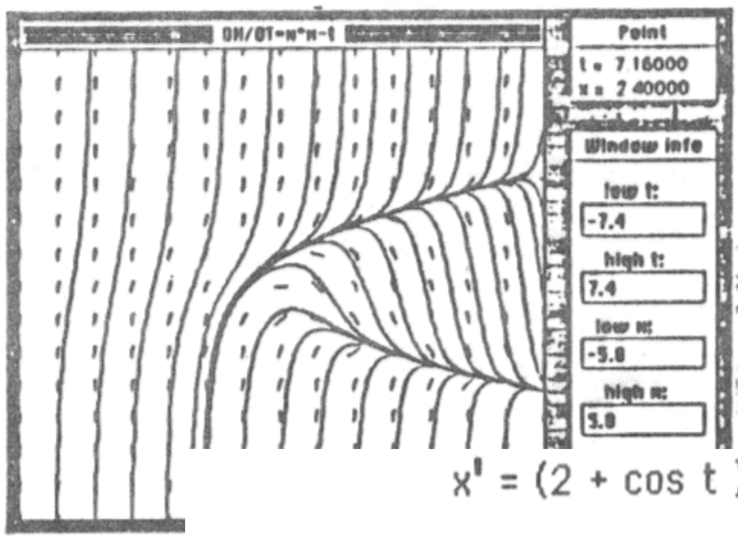
The influence of ***computers*** and ***informatics*** on ***mathematics*** and its ***teaching***

Benoit Mandelbrot
(1924 – 2010)
October 14th, 2010



The influence of *computers* and *informatics* on *mathematics* and its *teaching*

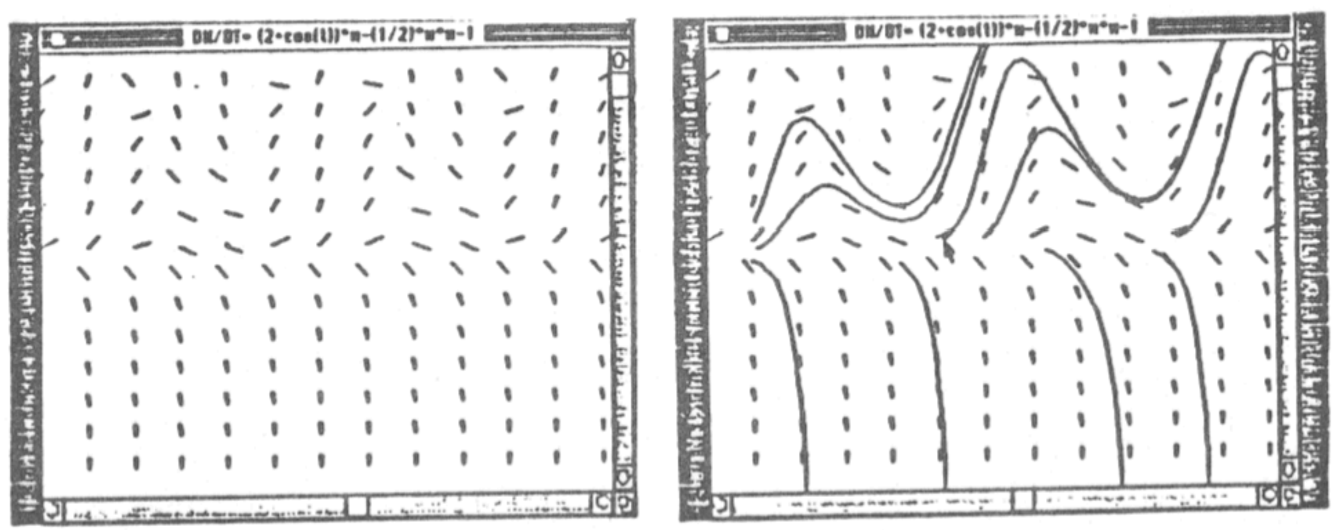
$x' = x^2 - t$



John Hubbard & Beverly West
(Cornell)

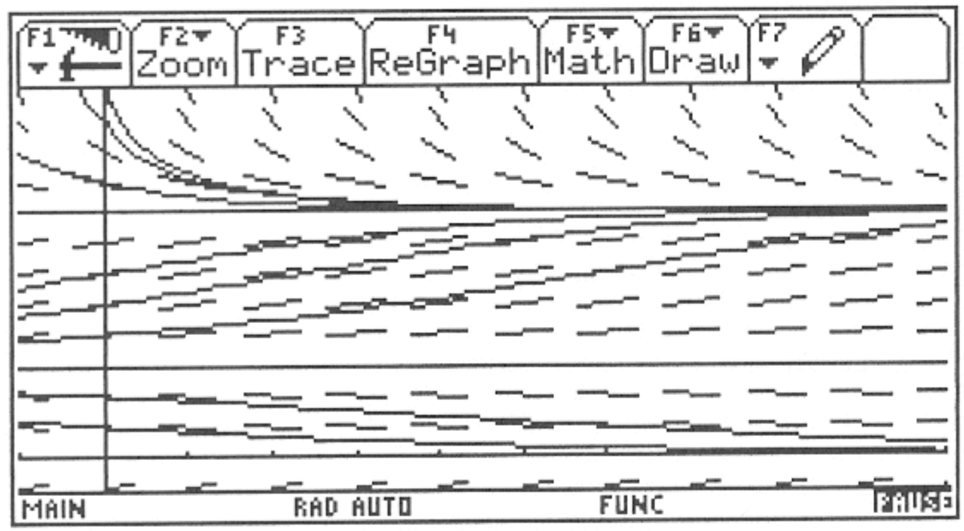
$x' = (2 + \cos t)\, x - (1/2)\, x^2 - 1$

slope field
(or *direction field*)
for a differential equation



The influence of *computers* and *informatics* on *mathematics* and its *teaching*

11th ICMI Study
(1998)



TI-92

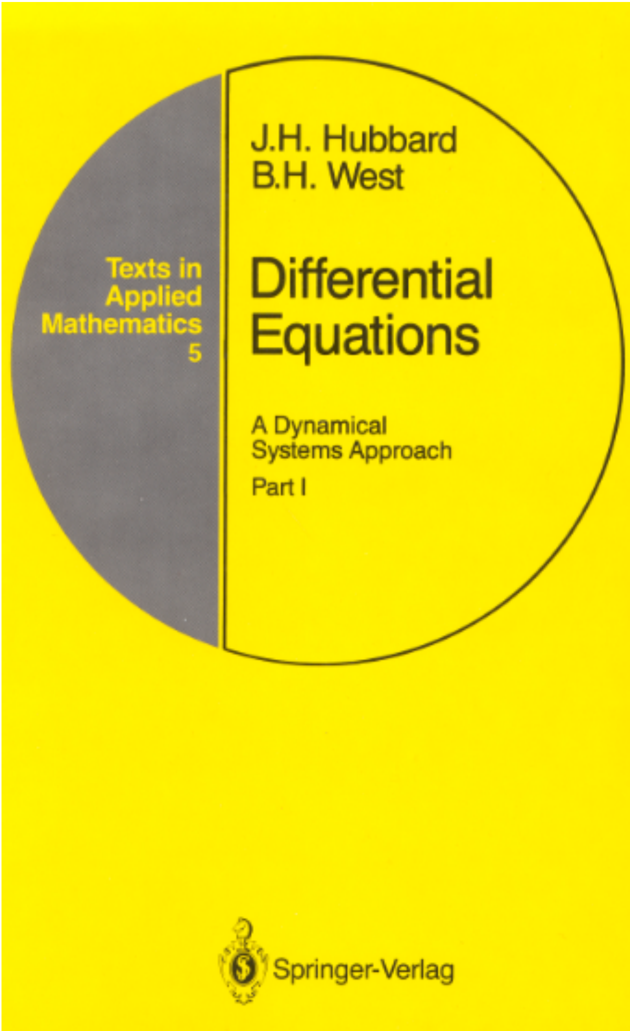
Figure 1 slope field program with slopes and several approximations
$dy/dt=0.3y(1-y/8)(y/3-1)$, $y(0)=1, 2, 3, 4, 5, 6, 8, 9, 11, 12$

Rehabilitation of the qualitative study of differential systems through computer generated plots

1983

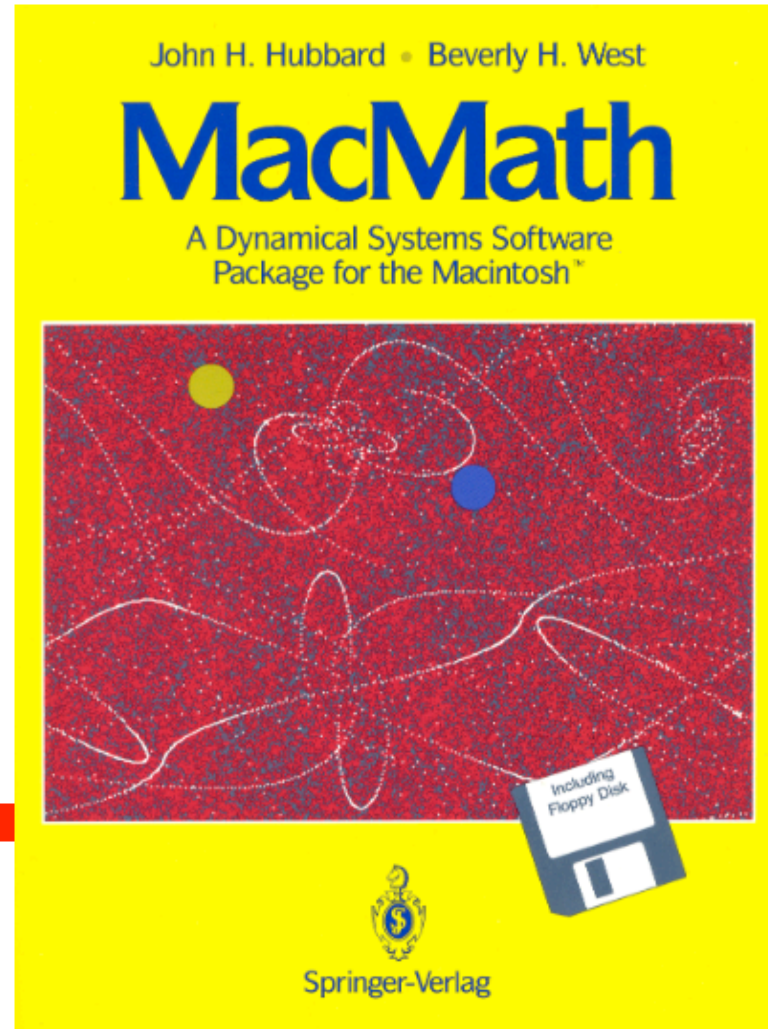
"L'apparition des micro-ordinateurs munis de bons écrans graphiques ou de tables traçantes a bouleversé les données du problème [analyse qualitative et graphique] en multipliant les possibilités d'observations et en permettant un gain de temps inespéré."

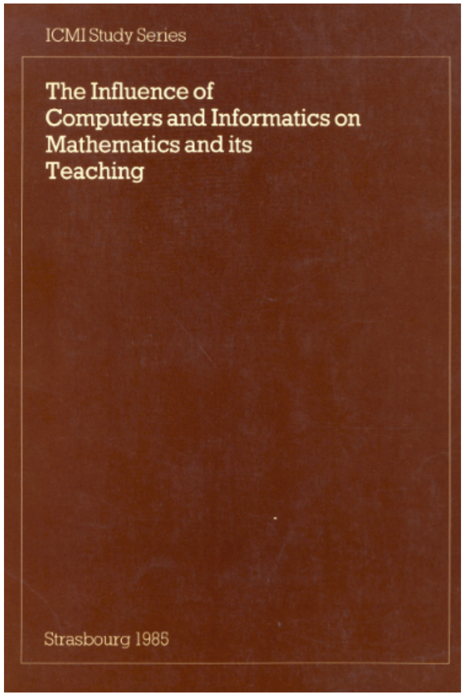
(1st ICMI Study, 1985)


J.H. Hubbard
B.H. West
Texts in Applied Mathematics 5
Differential Equations
A Dynamical Systems Approach
Part I
Springer-Verlag

1991

1992


John H. Hubbard • Beverly H. West
MacMath
A Dynamical Systems Software Package for the Macintosh™
Including Floppy Disk
Springer-Verlag



1st ICMI Study (1985)

Introductory Calculus in 1990

a paper for the I.C.M.I. Symposium on Computers and Mathematics*

B.R. Hodgson (Université Laval)
E.R. Muller (Brock University)
J. Poland (Carleton University)
P.D. Taylor (Queen's University)

1) Introduction

In this article we propose ways in which the introductory Calculus curriculum might respond to the recent and rapidly changing computer resources. Our purpose is not to describe how such computer resources might be used most effectively in the learning of the Calculus but rather to examine the impact of the existence of such resources as computer programs to perform differentiation and definite and indefinite integration.

Our main points are

- it is counterproductive to train our students to perform calculations that they know a microcomputer can do far more accurately and quickly;

- consequently a major reorientation in the style and content of the introductory Calculus course is needed, away from the performance of algorithms and towards a more meaningful and thoughtful experience;

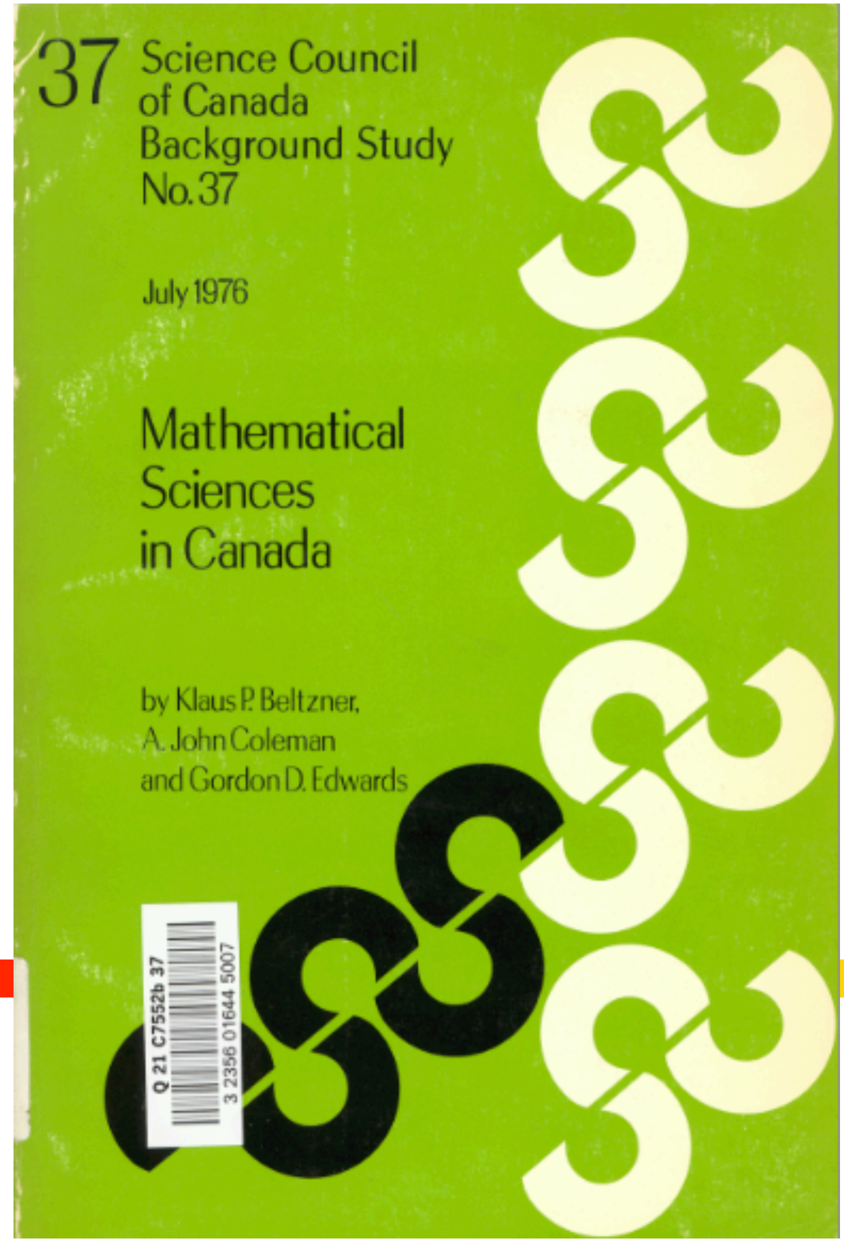
- the spirit of this change calls for presenting the Calculus as one of mankind's finest intellectual achievements, more valuable than ever in its recent applications, and demanding of more interactive classroom teaching.

In a sense, we are entering a golden age of mathematics teaching, in which the deemphasis upon paper-and-pen performance of algorithms frees us to teach in ways that respect what we each feel are the true goals of mathematics education.

"Most mathematics professors, when pressed, allege that their highest ambition in undergraduate teaching is to convey not specific content but rather a way of thinking. It was this way of thinking which we previously referred to as mathematizing."

A. John Coleman
1918 – 2010
(Sept. 30th, 2010)

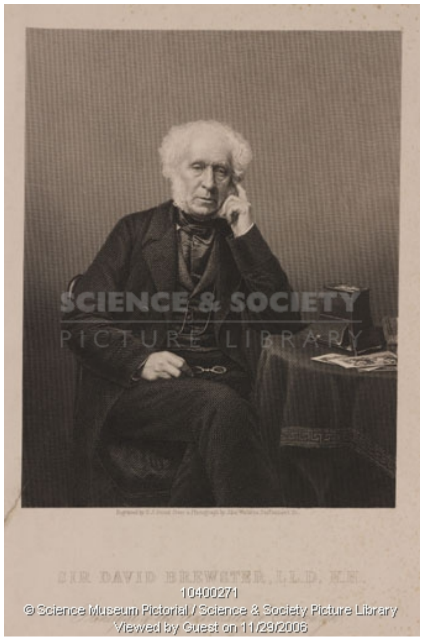
The "Coleman Report"



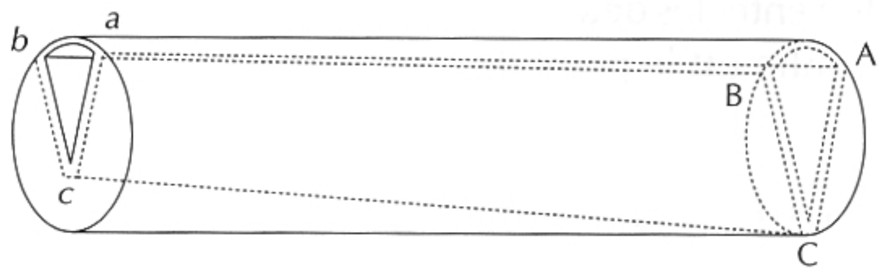
The "founding document"
of CMESG

A pet subject of mine…

The kaleidoscope



Sir David Brewster
(1781 – 1868)



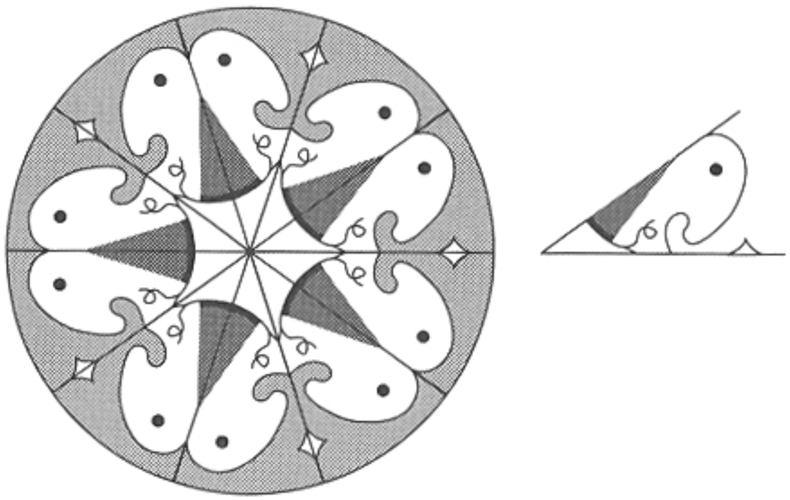
kalos (beautiful)

eidos (aspect)

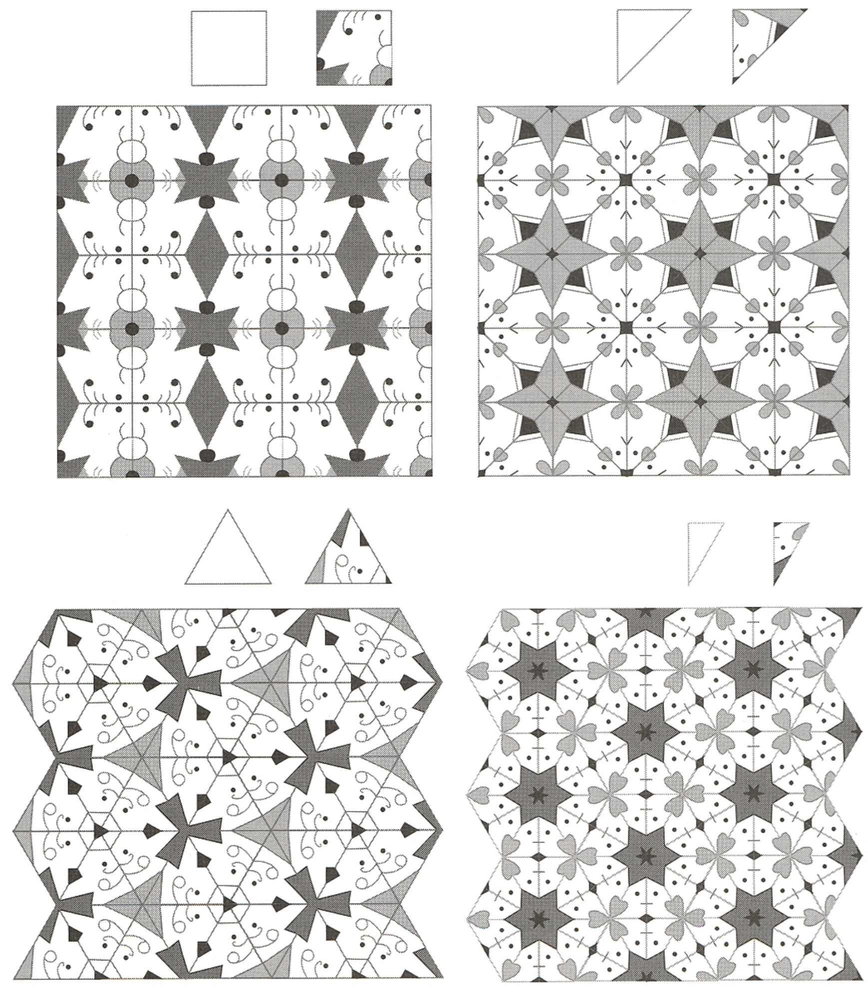
skopein (to look)

(1817)

(research on the polarisation of light)



rose-patterns



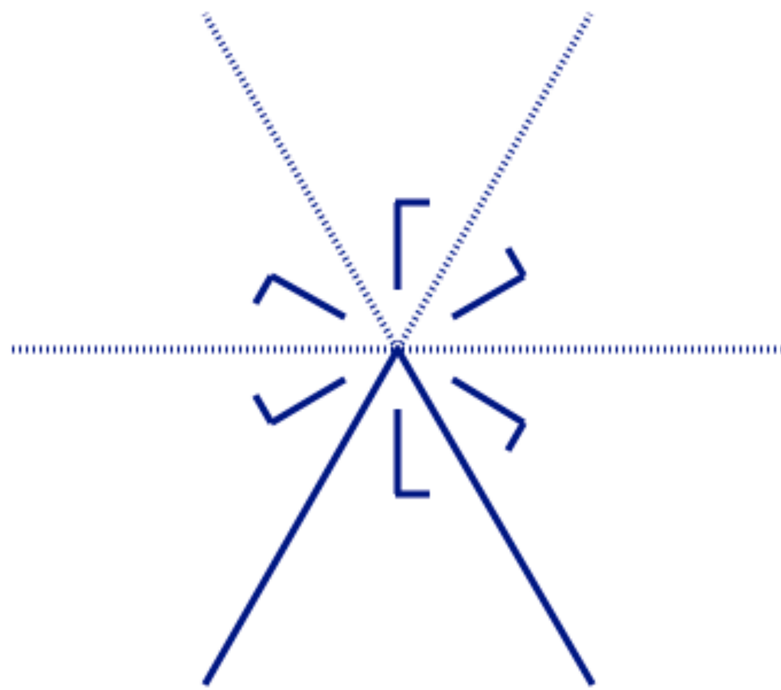
Brewster (1824)

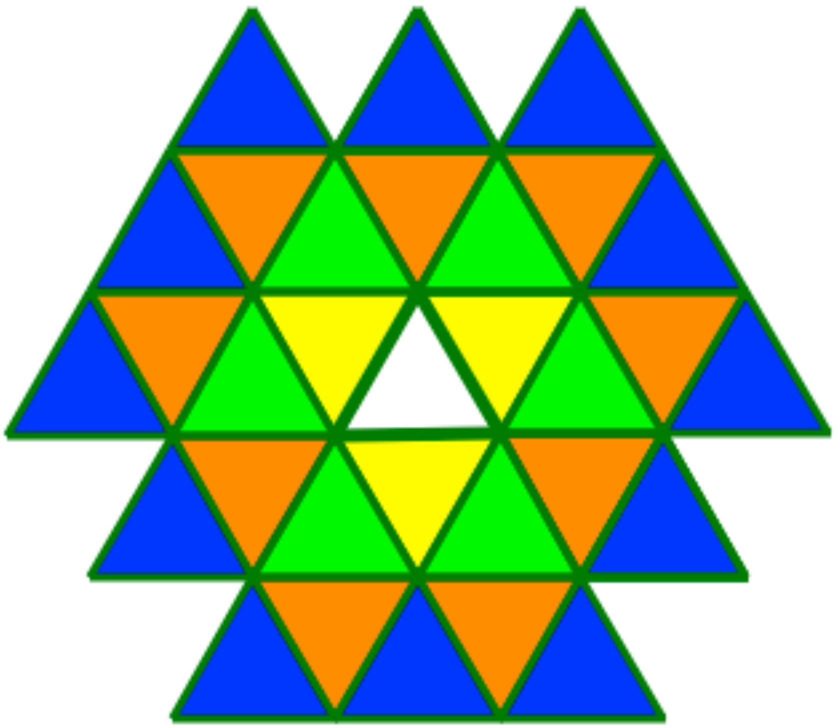
The Encyclopaedia Britannica

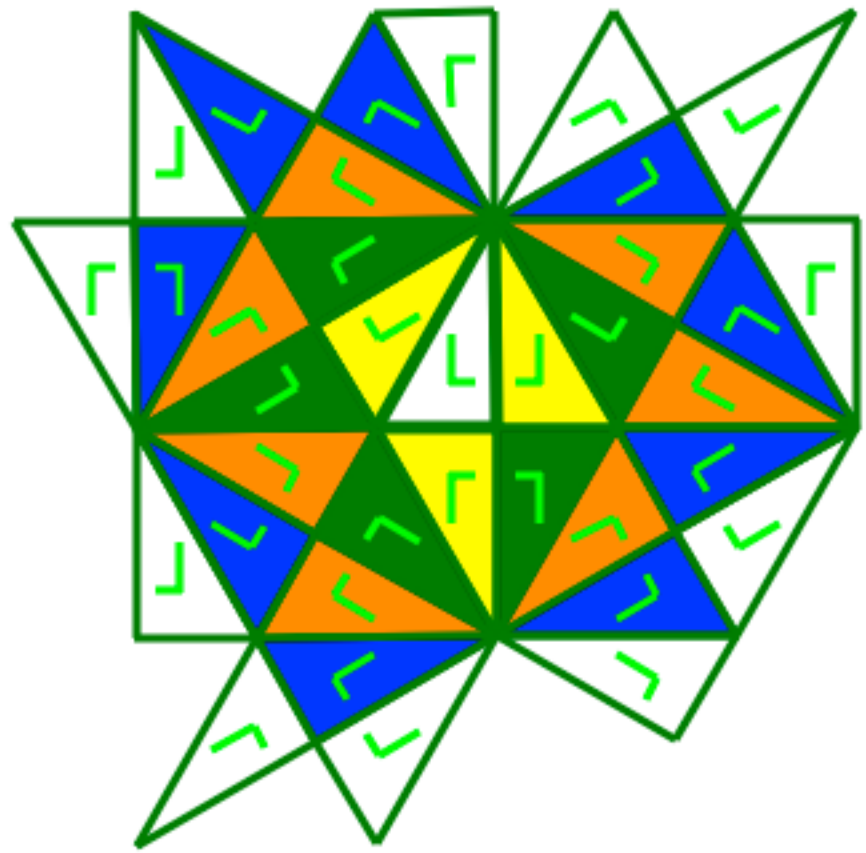
tiling-patterns

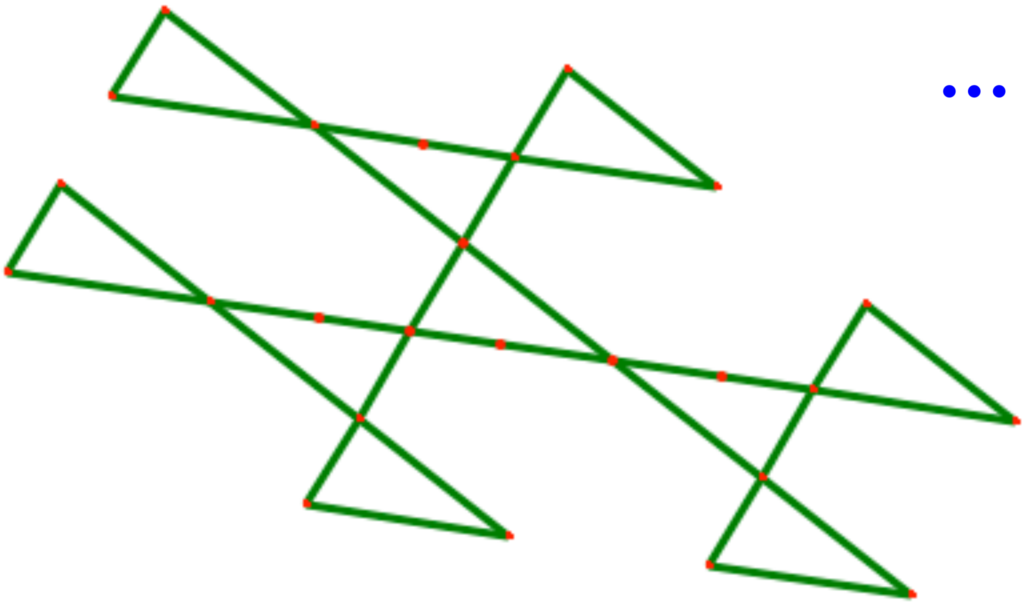
An instrument such as the kaleidoscope “renders obvious to the common observer what has hitherto been confined to the calculations of the mathematician.”

Charles Wheatstone (1802–1875)





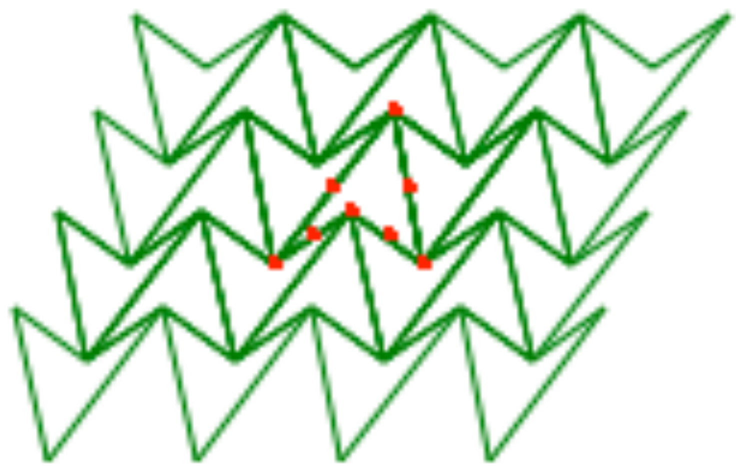




… reflection in a point
(vertex)

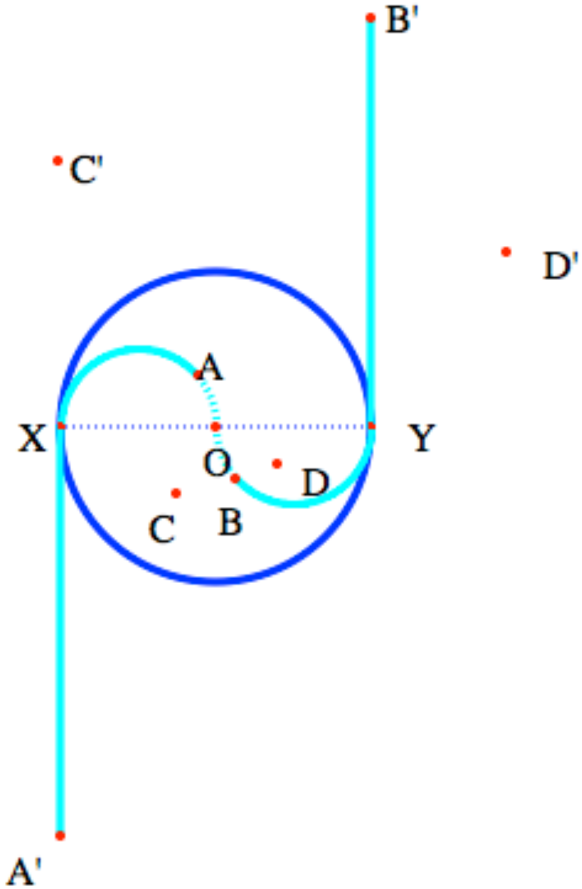
Fictitious kaleidoscopes!!!

***Reflection in a mirror* is replaced by…**

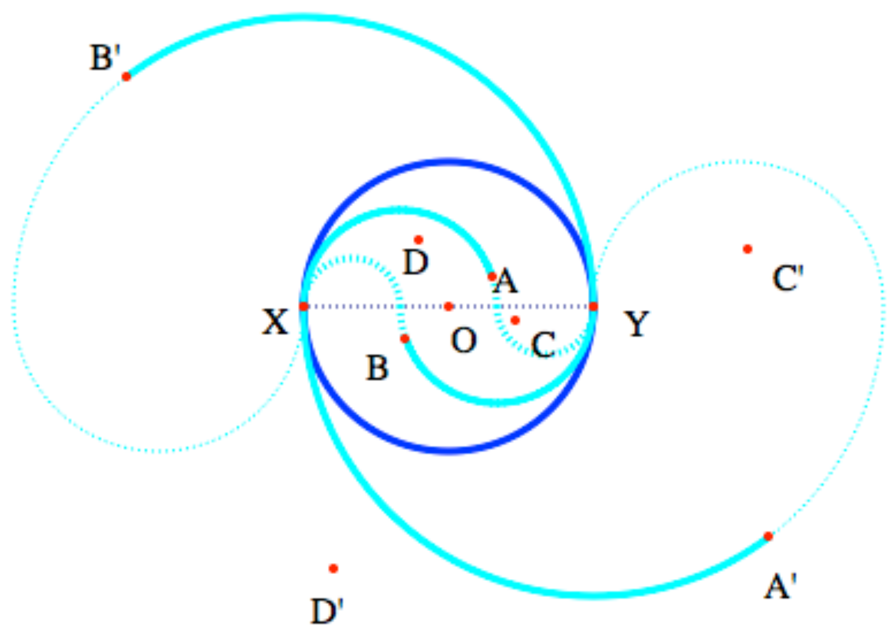


… reflection in a point
(midpoint)

… or by *inversion in a circle*

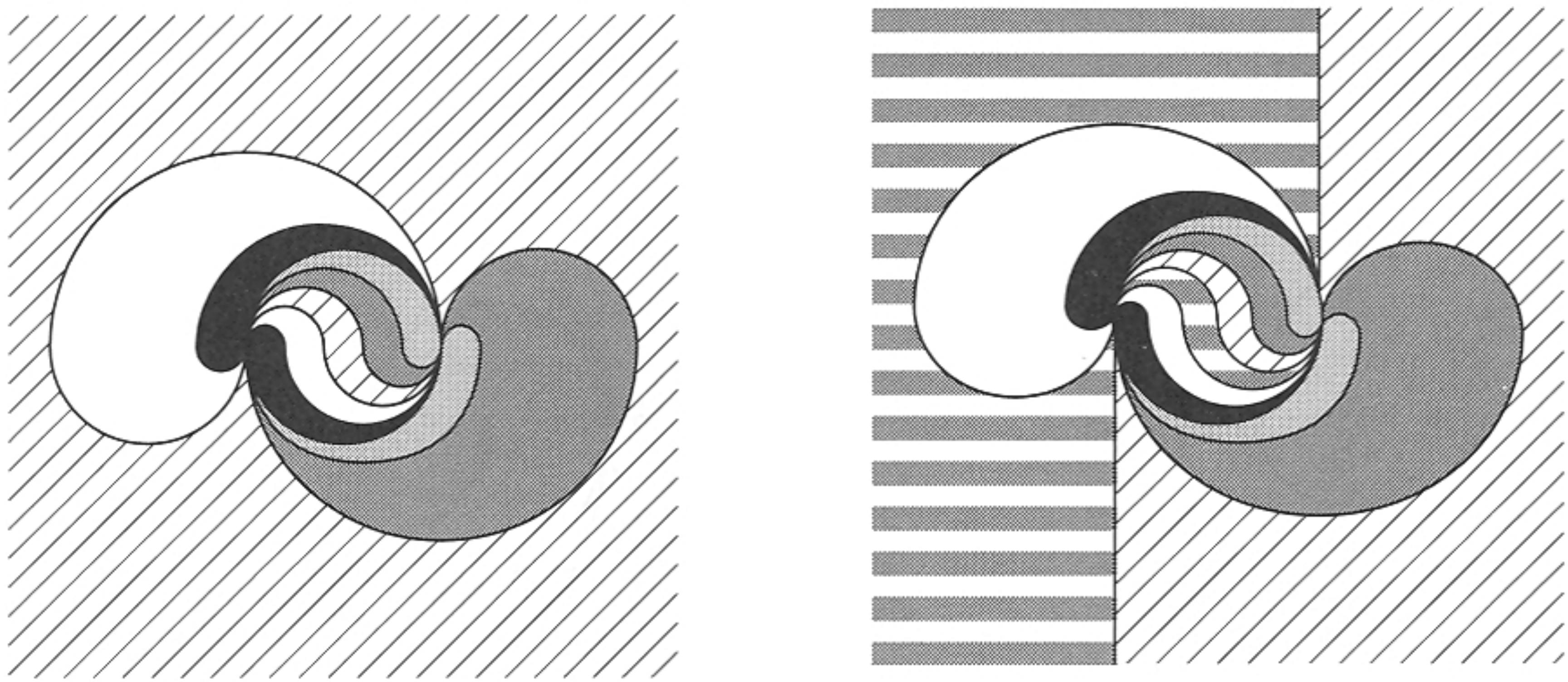


Virtual mirrors that the computer puts at our disposal



(plus horizontal reflection)

New “computer” tilings



division of the circle by “yin-yang” curves

The influence of computers and informatics on mathematics and its teaching: what about now?

UNIVERSITÉ LAVAL
Faculté des sciences et de génie

2222!;

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TEXAS INSTRUMENTS

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PLAN OF THE TALK

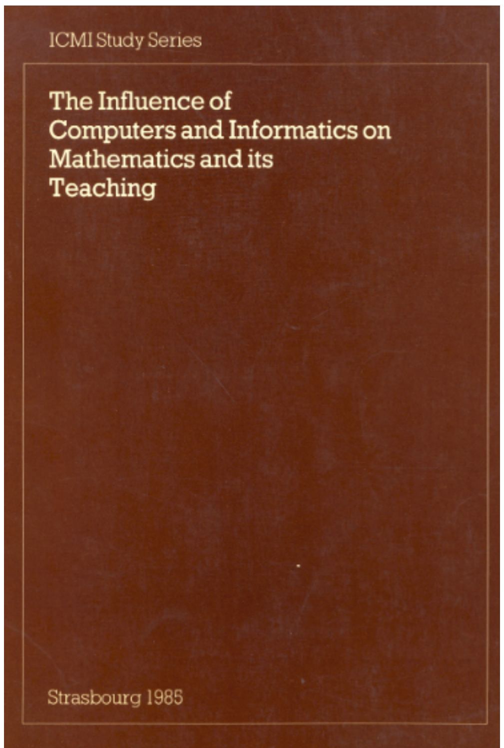
I- Mathematicians and mathematics education research

II- The influence of computers and informatics on mathematics and its teaching: a brief historical survey

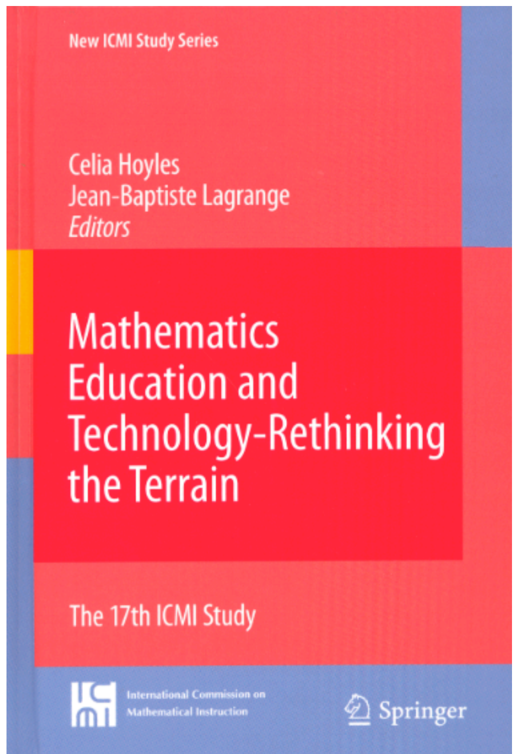
III- Technology in postsecondary mathematics education nowadays

III- Technology in postsecondary mathematics education nowadays

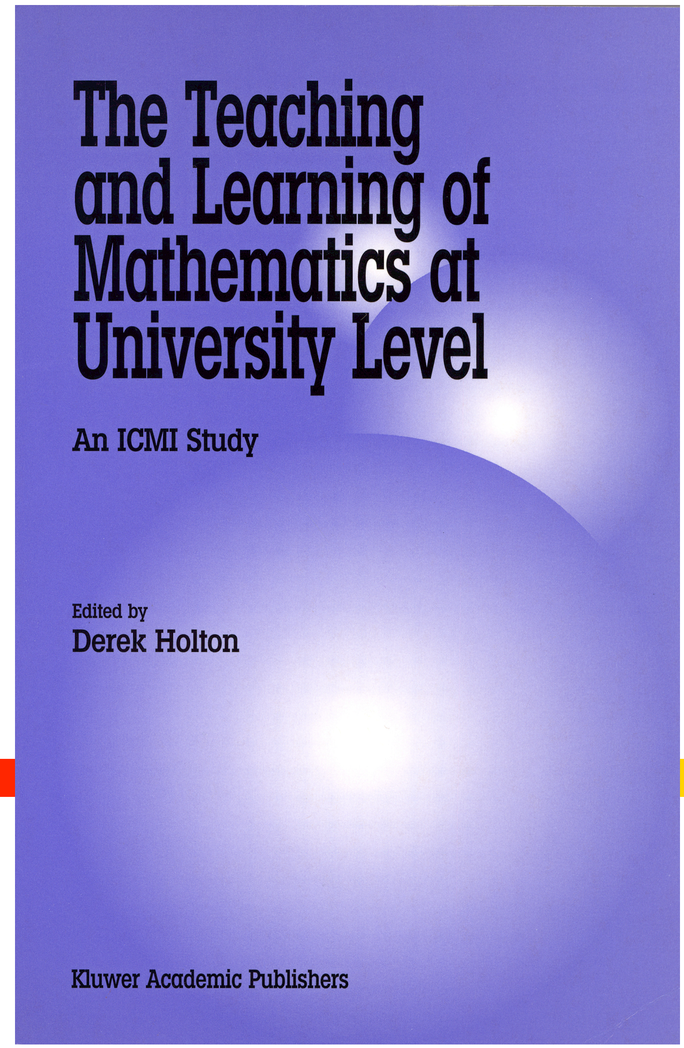
1st ICMI Study



17th ICMI Study



11th ICMI Study



Section edited by
Michèle Artigue and Joel Hillel

III- Technology in postsecondary mathematics education nowadays

Some difficulties remain…

"The introduction of computer technologies has had a dramatic impact on the place of the teacher in school. These technologies have contributed substantially to the evolution of the pedagogy. (…) They have also made the pedagogical contents sometimes more clear, more entertaining, more accessible."

"Letter" from a philosophy "cégep" teacher
Le Devoir, March 10, 2010

"New technology offers invaluable tools to the teacher. The problem does not come from the technology itself, but from the place given to it."

"The teacher is no more encouraged to become a specialist in a given domain of knowledge, but rather an expert in the use of new technologies." (…)

"New technology is not an end in itself, but a mean to reach an end: transmitting knowledge or competencies. Everywhere, in today's school, computer technology tends to replace the living word (« *la parole vivante* ») of the teacher."

"Letter" from a philosophy "cégep" teacher
Le Devoir, March 10, 2010

But there are some “success stories” …

… for instance the MICA programme (Brock University)

Mathematics Integrated with Computers and Applications

“MICA is a cutting edge mathematics program that teaches you how to use powerful combinations of mathematics and computers to solve sophisticated real world problems.”

“A four-year honours program that gives you a solid foundation in math and also teaches you the technology you need to know in order to apply what you’ve learned. You also have the option of specializing in education, pure mathematics, applied mathematics or statistics.”

- Muller – ICMI Study 11 (1998)
- Buteau & Muller -- ICMI Study 17 (2006)

Integration of technology

- **towards future professional needs**

  vg, expectations of employers or "state-of-the-art"

  - research mathematicians

    numerical methods, algebra, number theory, …

    (influence of computers on the mathematical activity *per se*, on the way to "do math")

  - users of mathematics

    engineers, scientists, …

  - primary or secondary school teachers

    secondary level: TI-83, TI-84, TI-Nspire

- **for purely pedagogical purposes**

Familiarisation with technology ***(demystification)***

- go beyond programming
- go beyond the mastery of a few basic tools and and the development of interfaces
- "low budget" accessibility to very powerful tools

  facility, conviviality of interface, … , complexity!
- what is the desirable "opacity" of technology?

  "black" box, or "grey", "white" … "pink"
- knowing when to "leave" technology

  John Mason, 1st ICMI Study:

  -- syndrome "Compute first and think second"

  -- "The hardest button to press is the off-button."

The crux of the matter: teachers!

- go beyond some "happy few" unwavering enthusiasts

  difficulty even within the math dept of ONE university

  and even more in a network (schools, "cégeps", …)

- development of specific knowledge and of a "sensibility" on three grounds:

  mathematics, "informatics", "didactics"

  -- *the weak link*

  -- *"short training" does not work*

- knowing to function in a context of diminution of certain *calculation* skills and of strengthening of *interpretation* and *approximation* skills

A warning

"technical / conceptual" opposition
("techno activist" discourse)

"The use of technology, by relieving the student of a certain technical labour, directly provides her with an access to a conceptual activity."

It depends…

A few "engaging questions"

- *How to help postsecondary teachers prepare themselves to work in a technological context?*
  With what support are they actually provided?
- *How is mathematics really changed by technology?*
  (in the light of a few decades of experimentations)
- *Does math education research bring robust arguments supporting the use of technology in postsecondary mathematics teaching and learning?*
- *What about the fact that technology allows students to use mathematical concepts before mastering computational techniques?*
- *And what about "new" environments?*
  ("e-exercises bases", etc.)

By way of conclusion…

I dream…

… of a framework where technology is used so to support a teaching and learning environment where HUMAN TO HUMAN interactions occupy the central place!

Thanks!